MECHANICS OF FLUIDS

Lecture 1 – Introduction

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Note

- All the art-work contents of this lecture are obtained from the following sources, unless otherwise stated:
  - *Fluid Mechanics, 8th edition, Frank M. White, McGraw-Hill, 2016.*
  - *Fluid Mechanics: Fundamental and Applications, 3rd edition, Yunus A. Cengel, John M. Cimbala, McGraw-Hill, 2014.*

Basic concepts (dimensions and units)

- Primary dimensions

Primary dimension	SI unit	BG unit	Conversion factor
Mass $\{M\}$	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length $\{L\}$	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time $\{T\}$	Second (s)	Second (s)	1 s = 1 s
Temperature $\{\Theta\}$	Kelvin (K)	Rankine (°R)	1 K = 1.8°R

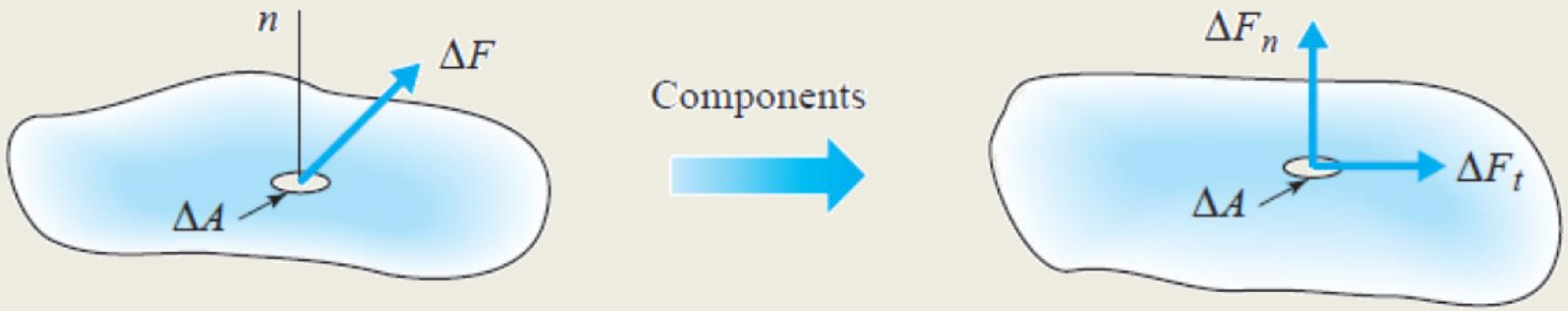
1 slug = 32.174 lbm

- gc, for converting mass to force

$$1\,N = 1\,kg \times 1\,\frac{m}{s^2} \qquad 1\,lbf = 1\,lb \times 32.174\frac{ft}{s^2} \qquad 1\,lbf = 1\,slug \times 1\,\frac{ft}{s^2}$$

Basic Concepts (stress)

- Force divided by surface is stress
  - *Normal*
  - *Shear*



$$\sigma_n = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A}$$

Normal stress

$$\tau = \lim_{\Delta A \to 0} \frac{\Delta F_t}{\Delta A}$$

Shear stress

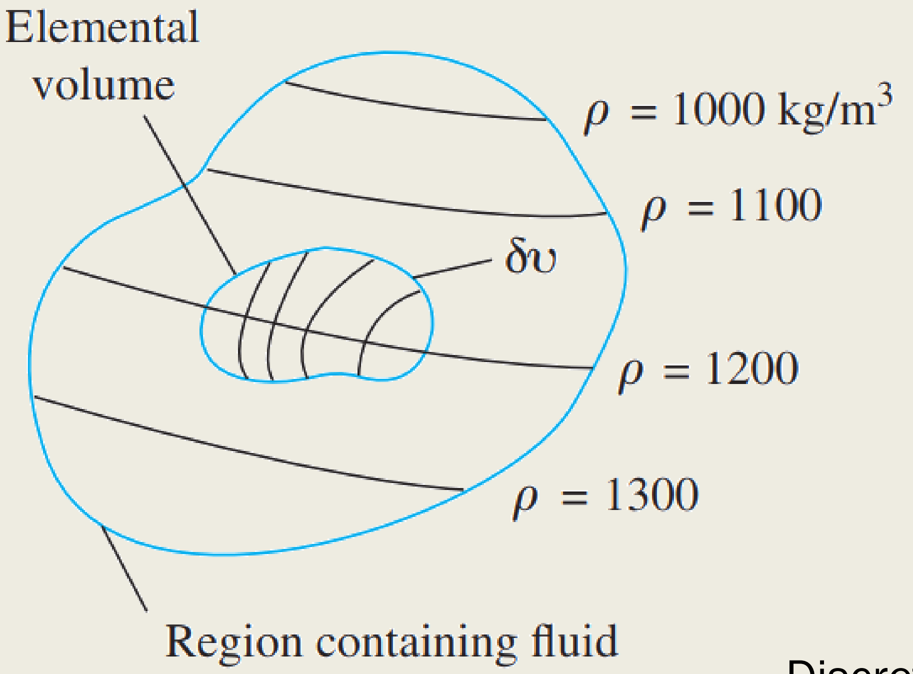
Basic Concepts (stress)

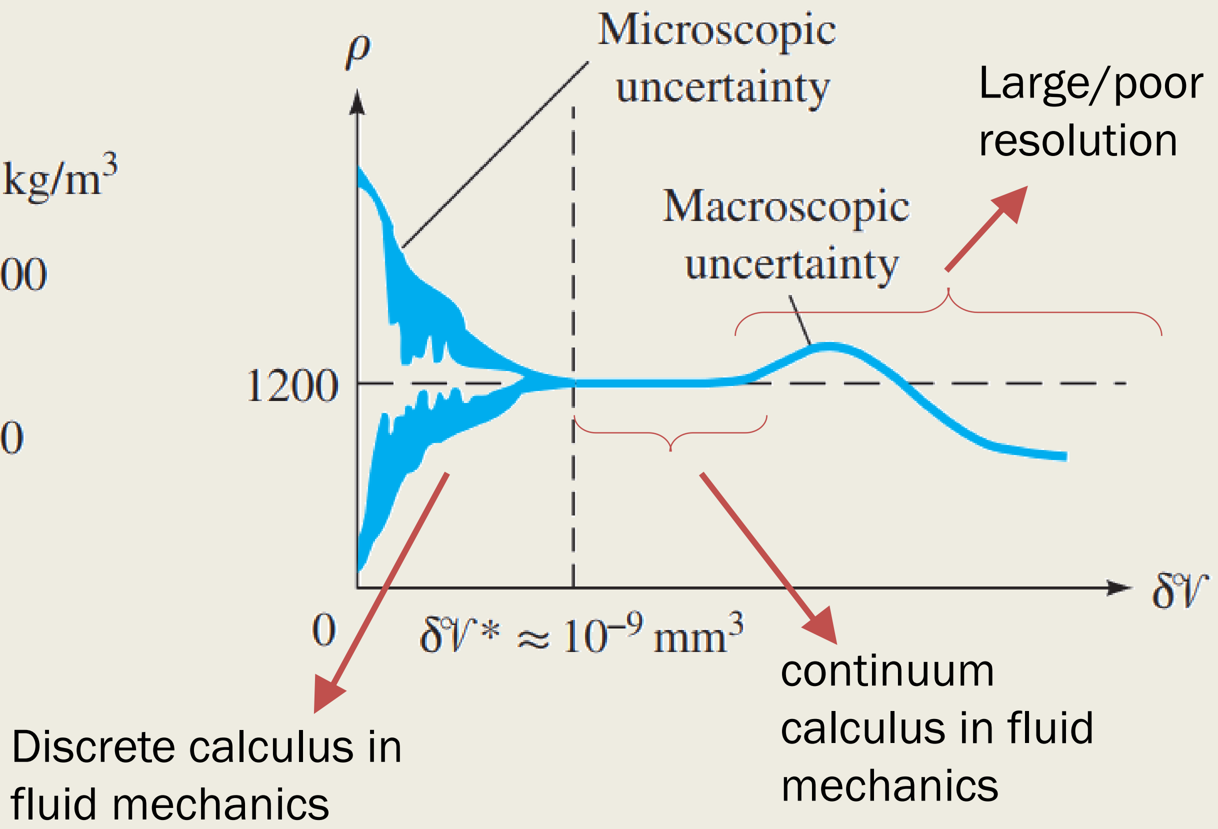
- Fluids (in physics)
  - *Gas: unrestricted motion of molecules which has no definite volume*
  - *Liquid: restricted motion of molecules which occupy almost constant volume*
- What do we mean by fluid in mechanics of fluids?
  - *Any liquid and gas that move under the action of shear stress, no matter how small the shear stress is.*

Basic Concepts (continuum vs. discrete)

- Consider the flow of fluid, density is defined as:

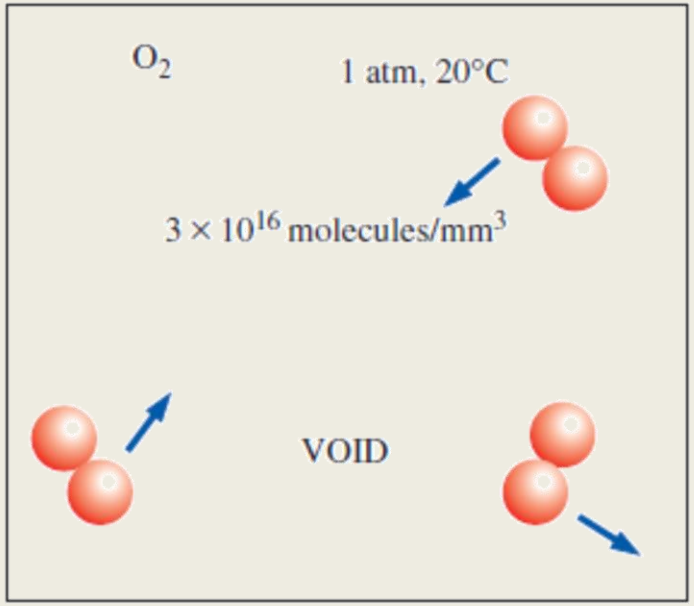
$$\rho = \lim_{\delta \mathcal{V} \to \delta \mathcal{V}^*} \frac{\delta m}{\delta \mathcal{V}}$$





Basic Concepts (continuum vs. discrete)

- For all liquids and most of gases the critical volume ($\delta \mathcal{V}^*$) is around 10^{-9} mm^3(each edge is around 1 microns).



- Mass density {M/L^3}:

Air (1 atm, 15 °C)	Water (1 atm, 4°C)
1.225 kg/m^3	1000 kg/m^3
0.0765 lb/ft^3	62.43 lb/ft^3

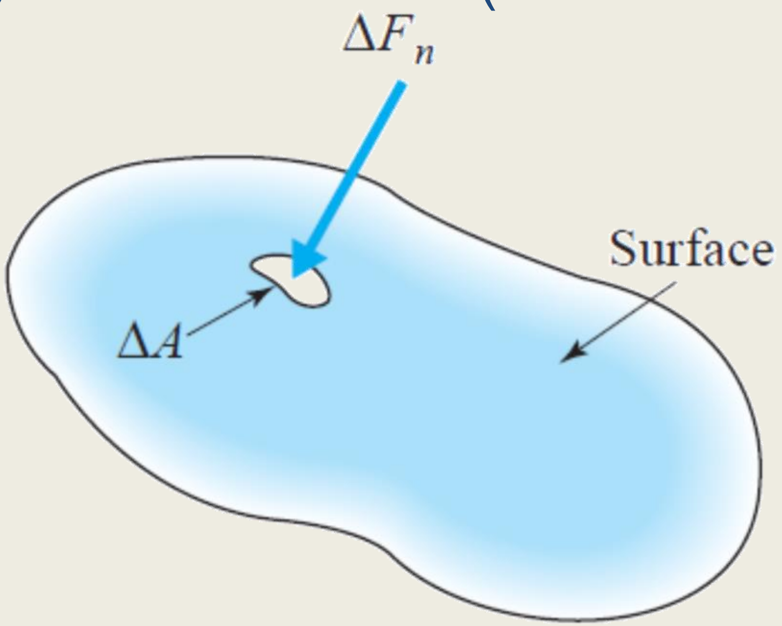
Basic Concepts

- Gas density varies with pressure and temperature for pure gases.
  - *In fluid mechanics calculations:*
    - For trivial calculations of sub-sonic flows (flows with low speed), it is assumed to be constant
    - For detailed calculations of sub-sonic flow, it is calculated by equation of state (EOS)
    - For near sonic flows it is always assumed inconstant.
- Liquid density varies with temperature only (is a poor function of pressure)
  - *In fluid mechanics calculations: it is usually assumed constant*
    -

Basic Concepts (pressure)

- Pressure results from compressive, normal forces (from fluid molecules) acting on the surface:

$$P = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A}$$



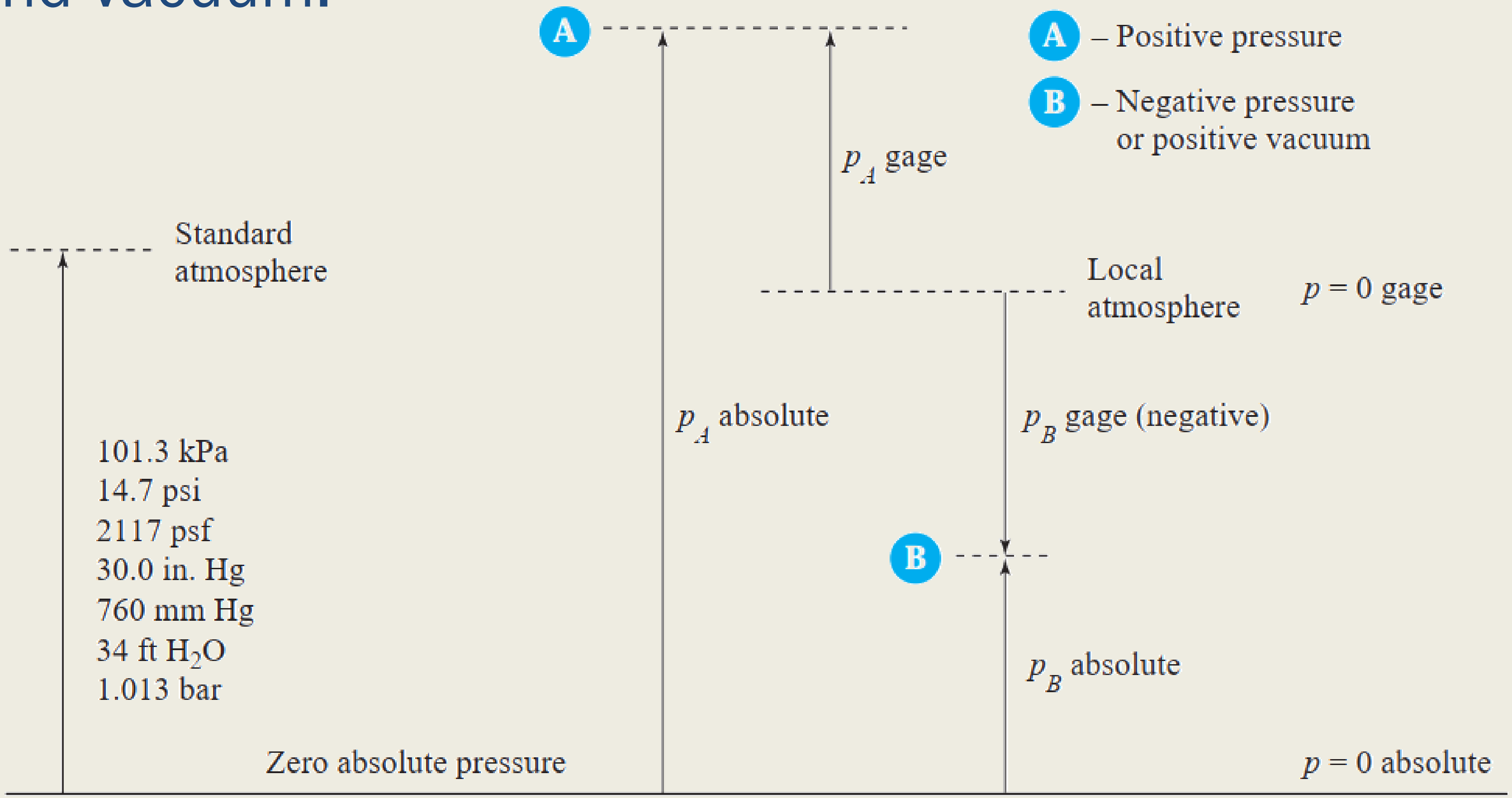
- Dimensions {M/(L.T^2)}
- Common units and values
  - *1 atm ≈14.7 psi = 101.325 kPa = 760 mmHg*
  - *1 psi = 144 psf*
  - *100 kPa = 1 bar*

Basic Concepts (pressure)

- Absolute, gage and vacuum:

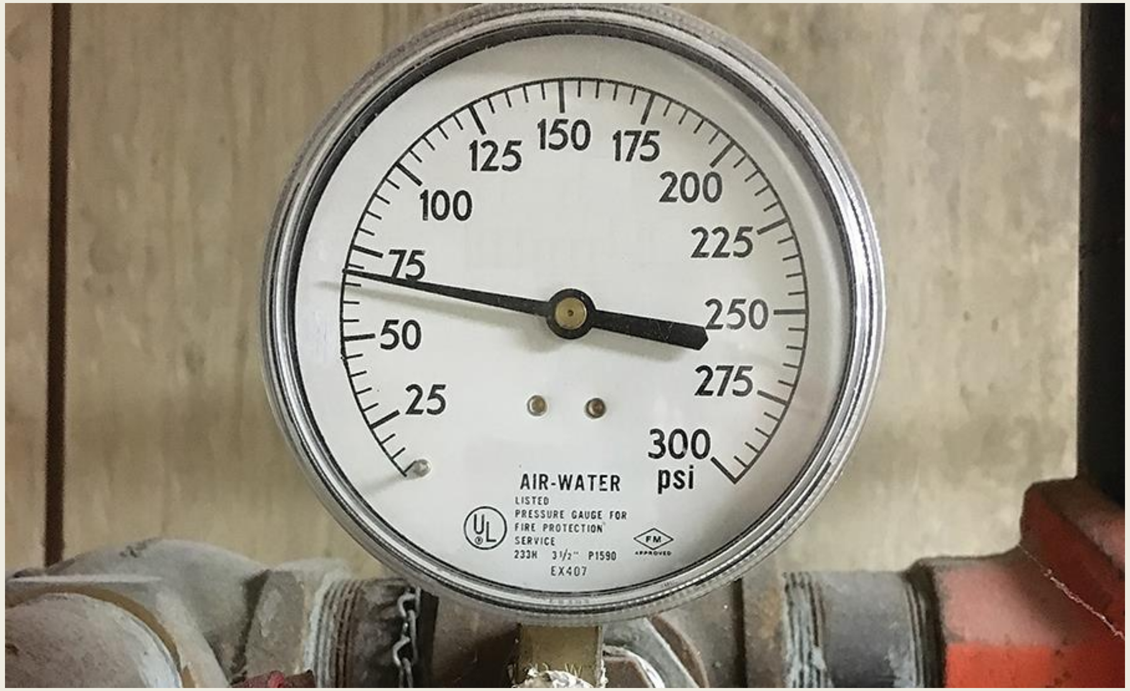
$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$



Source: mechanics of fluids, Potter et al. 4th edition, 2011.

Basic Concepts (pressure)



Source https://www.process-cooling.com/

Fluid Properties (thermodynamic props.)

- Specific weight: volumetric weight (weight divided by volume)

$$\gamma = \rho g$$

$$\gamma_{air} = (1.205\ \text{kg/m}^3)(9.807\ \text{m/s}^2) = 11.8\ \text{N/m}^3 = 0.0752\ \text{lbf/ft}^3$$

$$\gamma_{water} = (998\ \text{kg/m}^3)(9.807\ \text{m/s}^2) = 9790\ \text{N/m}^3 = 62.4\ \text{lbf/ft}^3$$

20 °C
1 atm

- Specific gravity:

$$SG_{gas} = \frac{\rho_{gas}}{\rho_{air}} = \frac{\rho_{gas}}{1.205\ \text{kg/m}^3}$$

Air at 20 °C, 1 atm

$$SG_{liquid} = \frac{\rho_{liquid}}{\rho_{water}} = \frac{\rho_{liquid}}{1000\ \text{kg/m}^3}$$

Water at 4 °C

Fluid Properties (viscosity)

- The internal stickiness of the fluid or the resistance of the fluid to start moving when it is influenced by a shear stress.

- Dimension {M/(L.T)}
- Common units and values:
  - *Gases: function of T and weak function of P*
  - *Liquids: function of T*
- Air (20 °C, 1 atm): 1.81×10^{-5} Pa.s
- Water (20 °C): 1 mPa.s

Fluid Properties (viscosity)

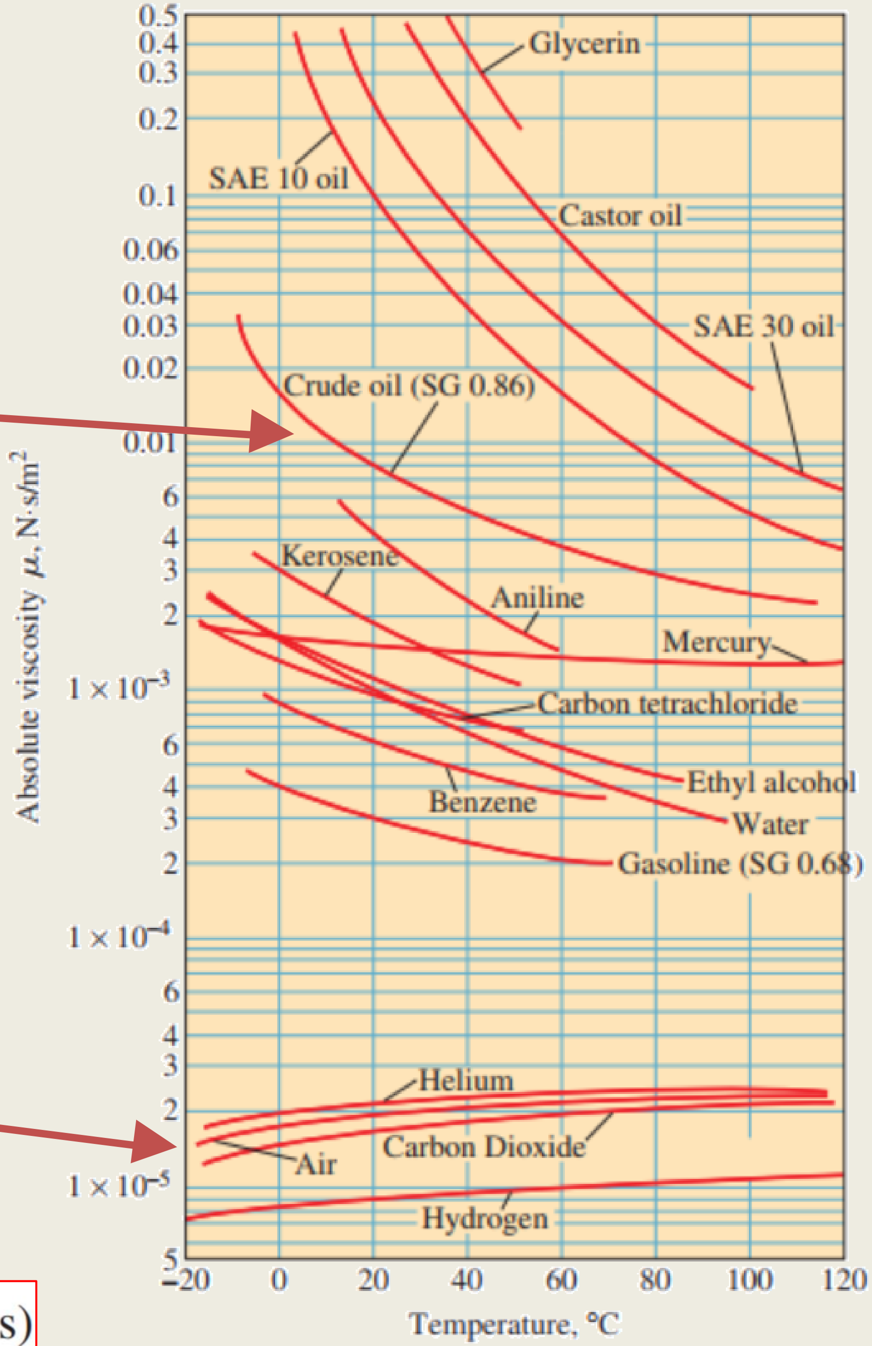
TABLE 2–3

Dynamic viscosity of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic Viscosity μ, kg/m·s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

$$\mu = a10^{b/(T-c)}$$

$$\mu = \frac{aT^{1/2}}{1 + b/T}$$



Remember: 1 cP = 1 mPa.s

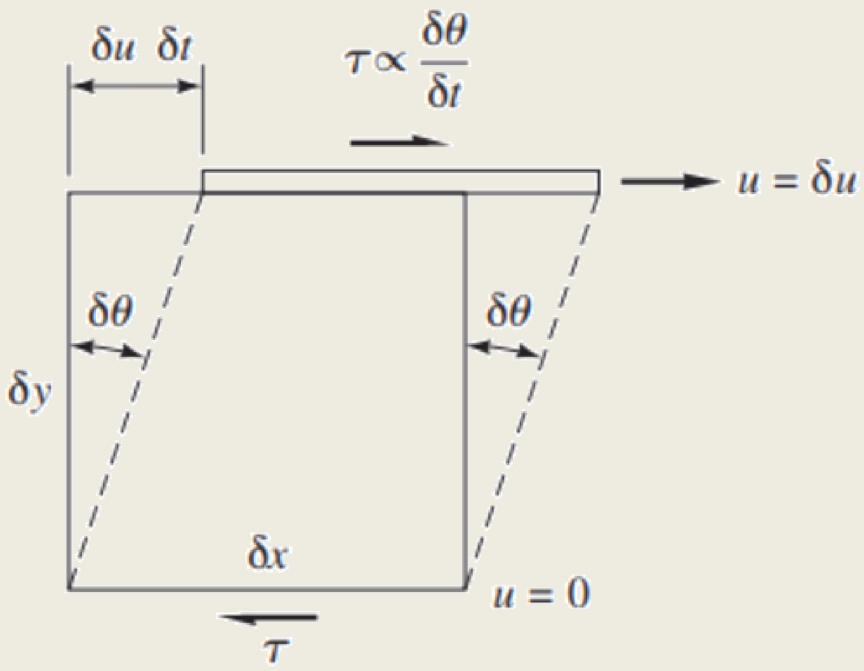
1 kg/(m · s) = 0.0209 slug/(ft · s)

Fluid Properties (viscosity)

- For common fluid such as water, oil, air

$$\underbrace{\tau}_{\text{Shear stress}} \propto \underbrace{\frac{\delta\theta}{\delta t}}_{\text{Strain rate}}$$

$$\tan \delta\theta = \frac{\delta u\ \delta t}{\delta y} \Rightarrow \frac{d\theta}{dt} = \underbrace{\frac{du}{dy}}_{\text{Velocity gradient}}$$



Fluid Properties (viscosity)

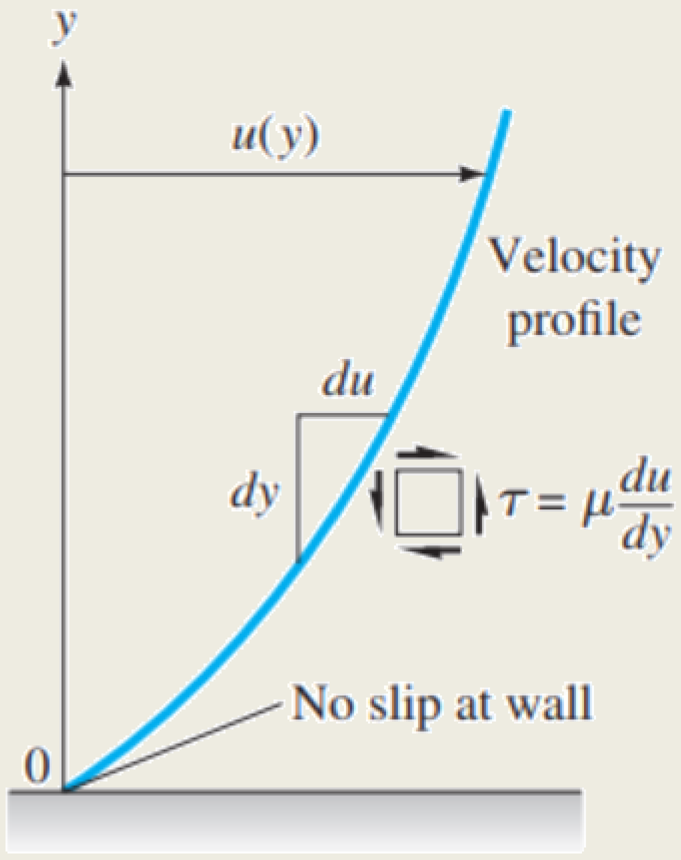
- For a Newtonian fluid (1D flow):

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy}$$

Dynamic viscosity or
Absolute viscosity

- Kinematic viscosity
  - *Dimension $\{L^2/s\}$*

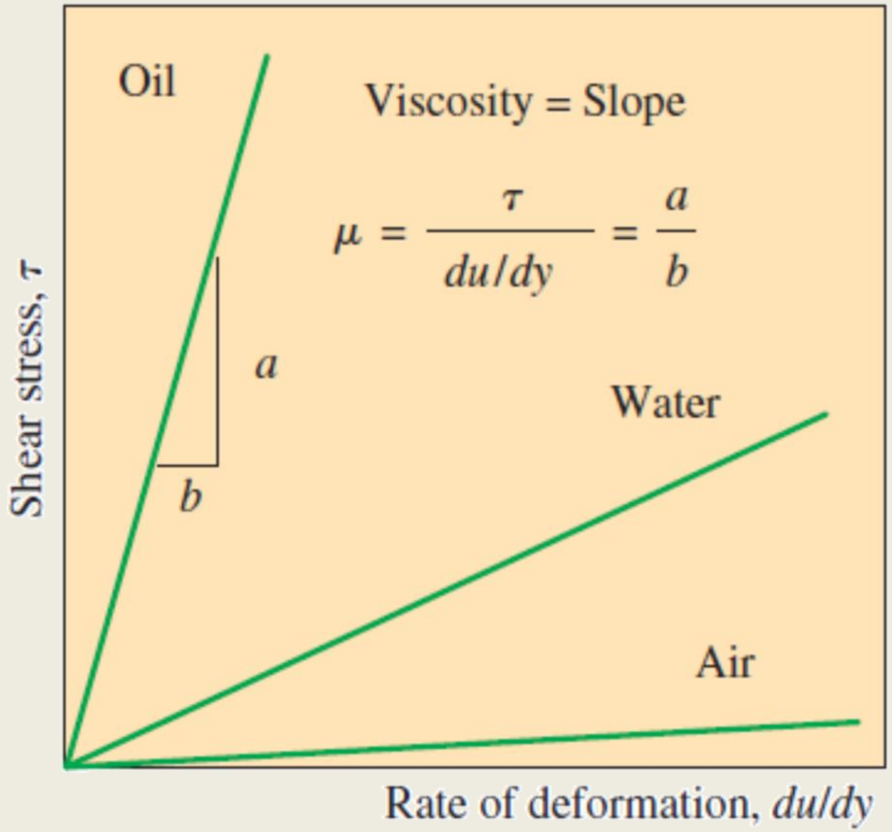
$$\nu = \frac{\mu}{\rho}$$



Sign convention:
From greater y to lesser y

Fluid Properties (viscosity)

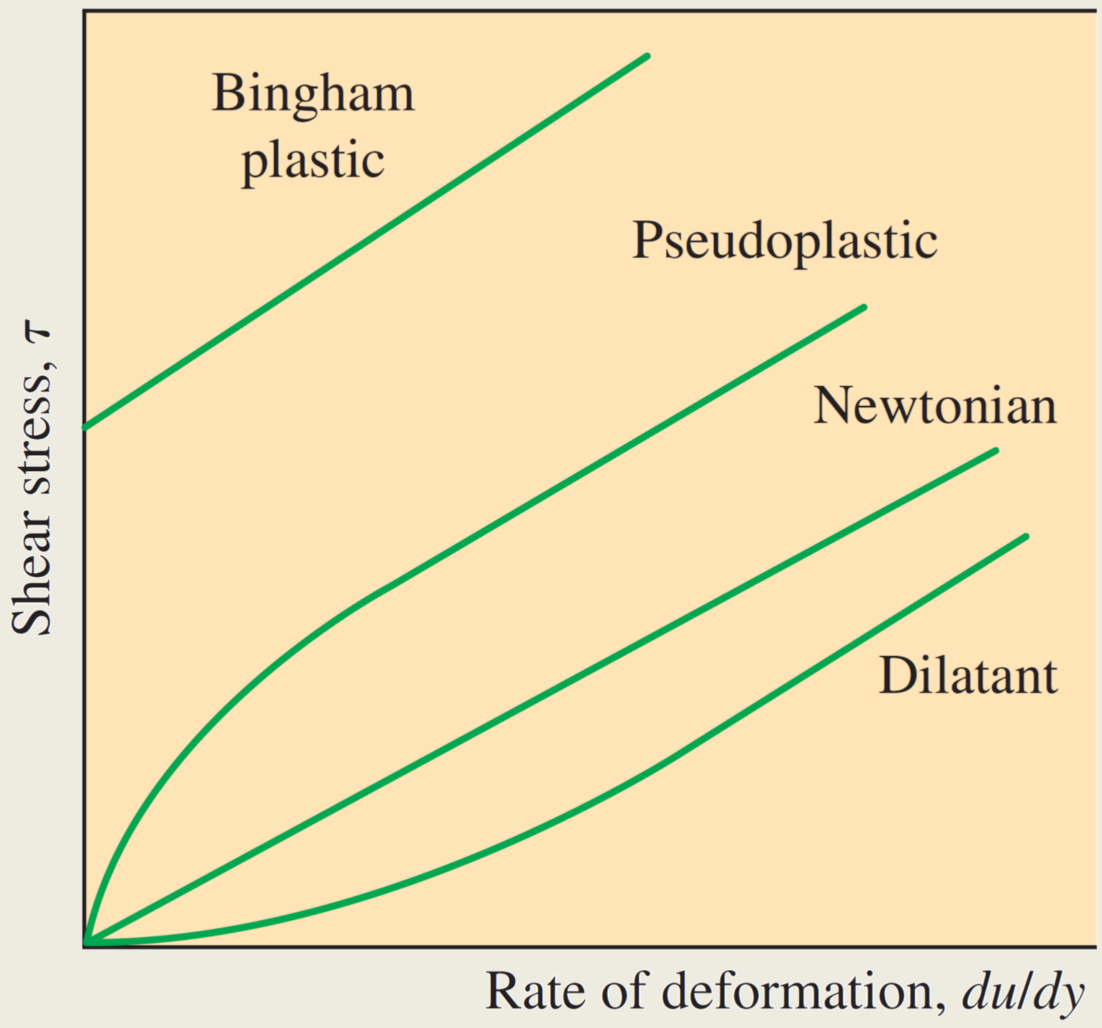
- For a Newtonian Fluid

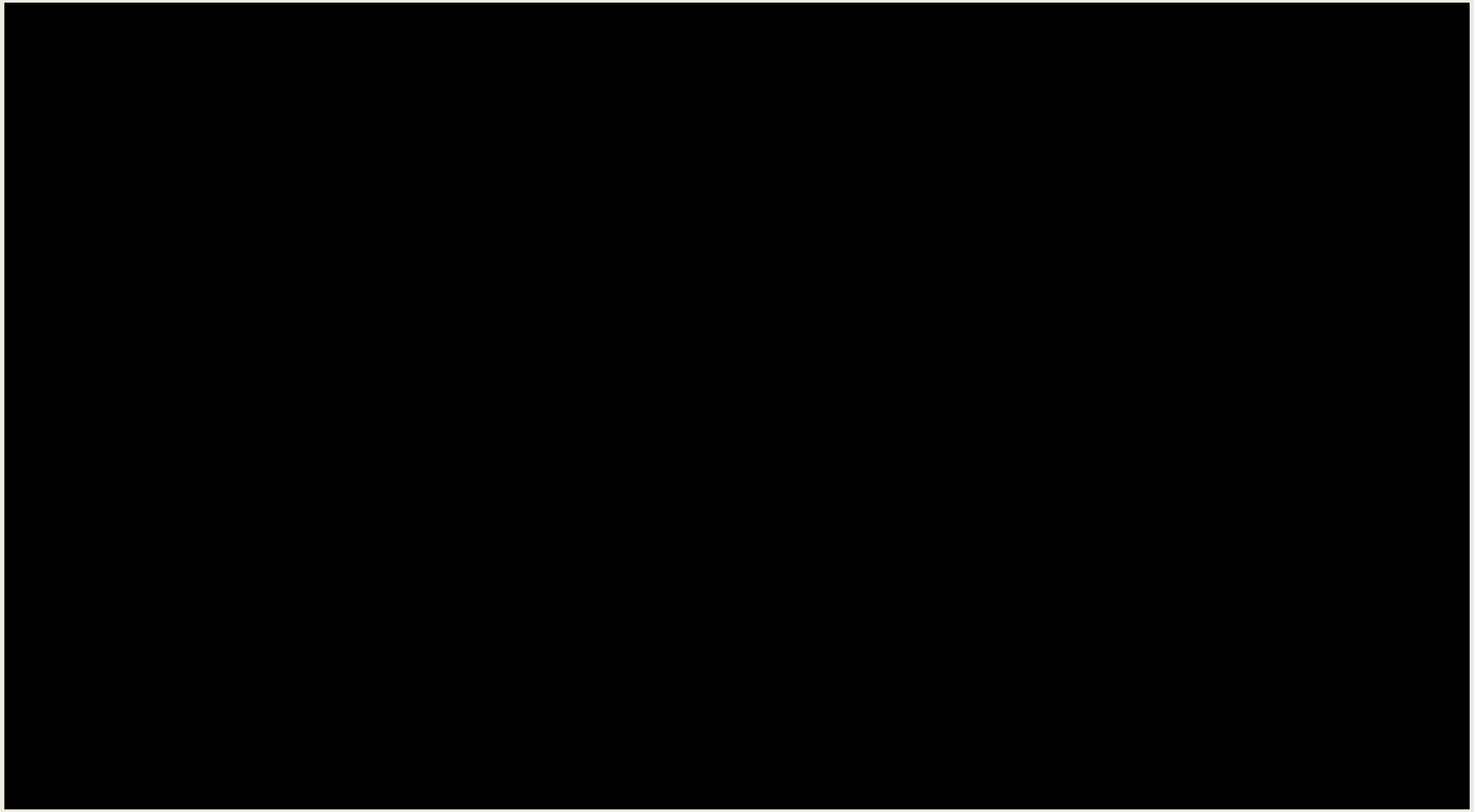


Fluid Properties (viscosity)

- Dilatant (Shear thickening fluid): Starch, sand suspension
- Pseudoplastic (Shear thinning fluid): some paints and polymer solutions
- Bingham plastic: toothpaste

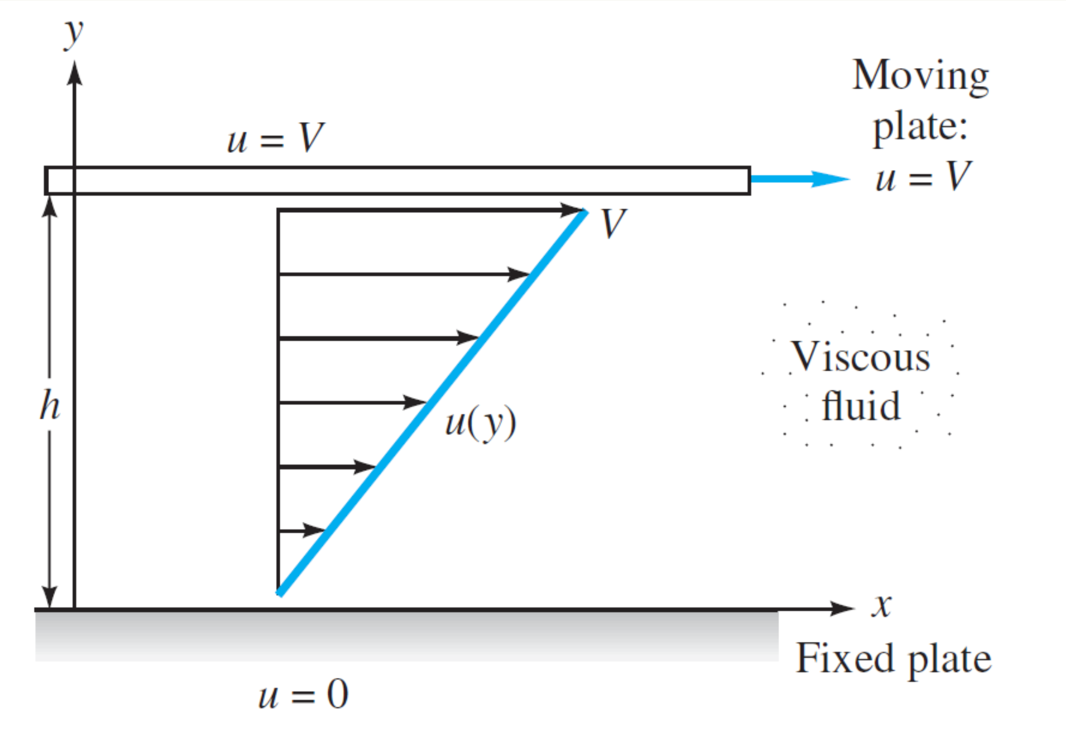
*Hereafter, we consider the fluid as **Newtonian**, unless otherwise stated!*





Flow Between Parallel Plates (laminar flow)

- If the plates are long enough, an steady one-dimensional shearing motion will be developed.



- Later it will be shown that:
  - *The acceleration is zero*
  - *Pressure gradient does not exist in the flow direction*
- At these conditions the shear rate is constant throughout the parallel layers of the fluid.

$$\tau = const$$

Flow Between Parallel Plates (laminar flow)

$$\frac{du}{dy} = \frac{\tau}{\mu} = \text{const}$$

Integration →

$$u = a + by$$

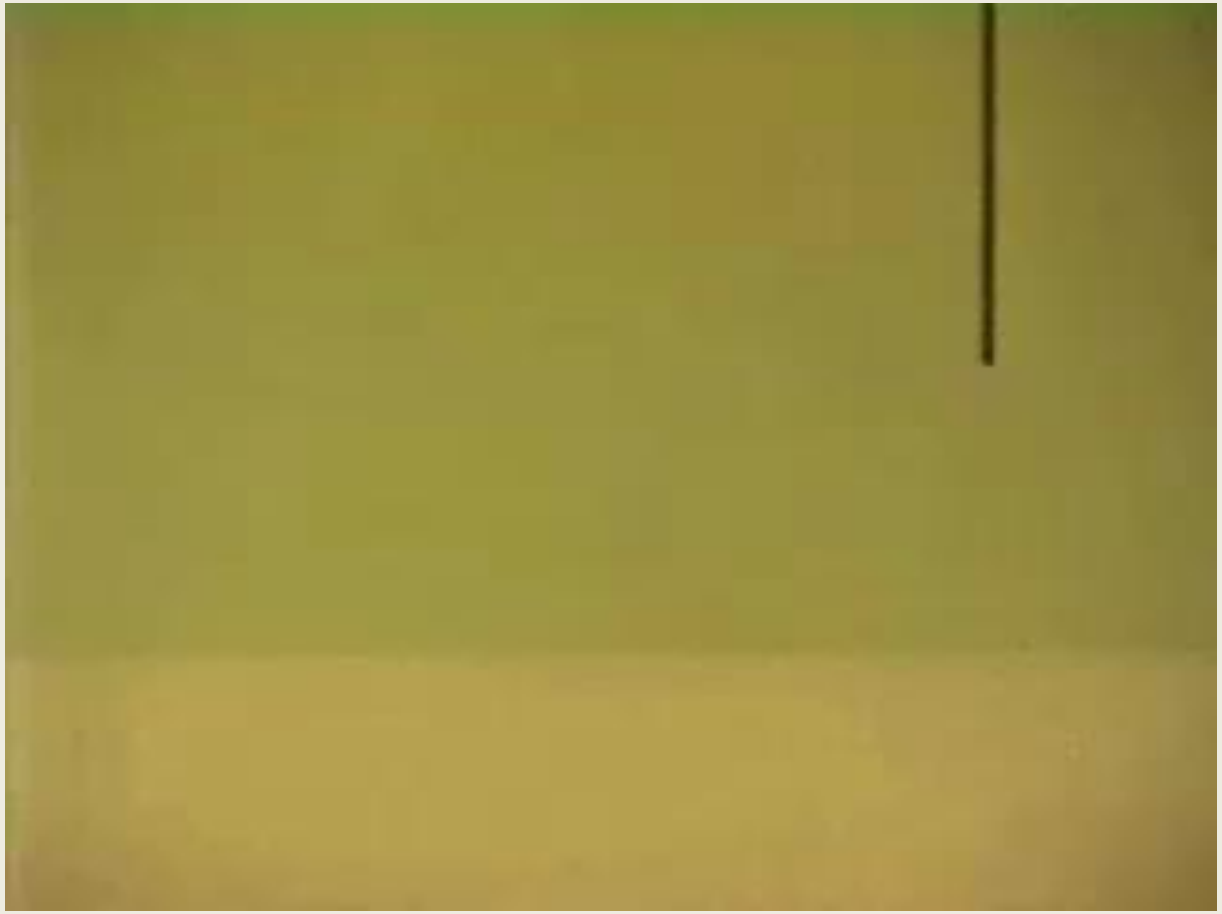
$$u = \begin{cases} 0 = a + b\,(0) & \text{at } y = 0 \\ V = a + b\,(h) & \text{at } y = h \end{cases}$$

No-slip boundary conditions

Applying boundary conditions →

$$u = V\frac{y}{h}$$

No-slip condition



Example

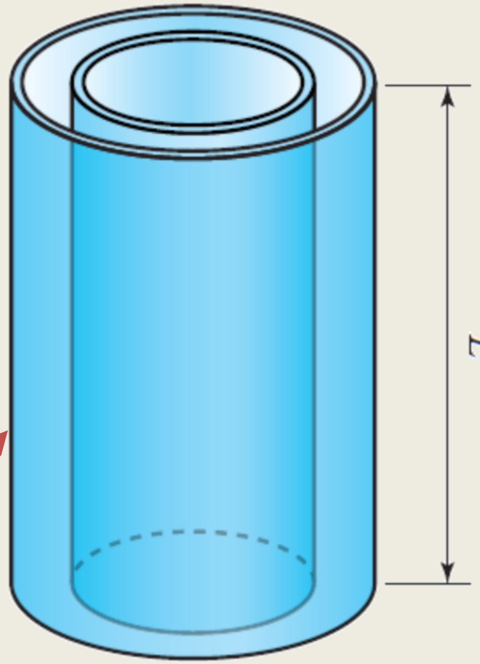
- Suppose that the fluid being sheared between two parallel plates is SAE 30 oil at 20 °C. Compute the shear stress in the oil if *V* = 3 m/s and *h* = 2 cm.

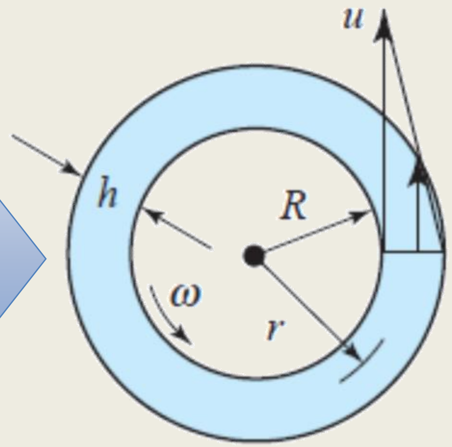
- **Solution:**
  - ***Assumptions:*** *steady condition, linear velocity, no-slip condition on plates*
  - ***Property:*** *SAE 30 oil viscosity at 20 ℃ is: 0.29 Pa.s*

$$\tau = \mu\frac{V}{h} = \left(0.29\,\frac{\text{kg}}{\text{m}\cdot\text{s}}\right)\frac{(3\text{ m/s})}{(0.02\text{ m})} = 43.5\,\frac{\text{kg}\cdot\text{m/s}^2}{\text{m}^2} = 43.5\,\frac{\text{N}}{\text{m}^2} \approx 44\text{Pa}$$

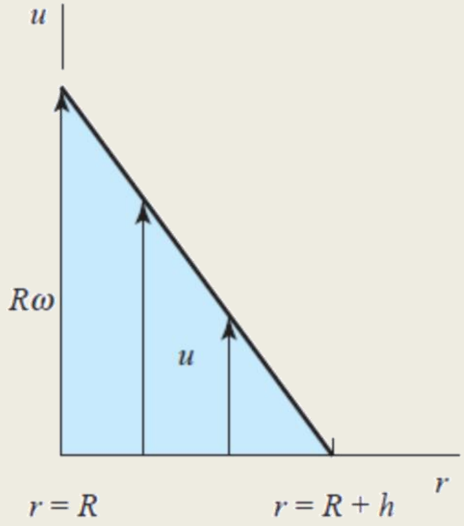
Viscometer (rotary)

- Obtain a formula that relates the viscosity to the torque?





h<< R
Neglect curvature



Inner cylinder rotates at ω

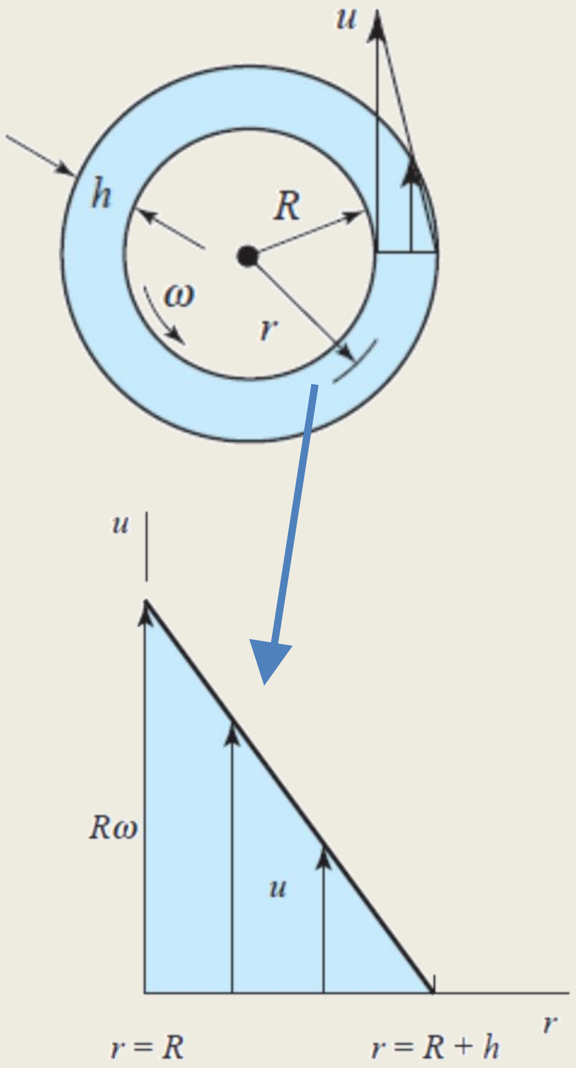
Source: tainstruments.com

Viscometer (rotary)

$$\tau = \mu \left| \frac{du}{dr} \right|$$

$$\left| \frac{du}{dr} \right| = \frac{\omega R}{h}$$

$$\begin{aligned} T &= \text{stress} \times \text{area} \times \text{moment arm} \\ &= \tau \times 2\pi R L \times R \\ &= \mu \frac{\omega R}{h} \times 2\pi R L \times R = \frac{2\pi R^3 \omega L \mu}{h} \end{aligned}$$



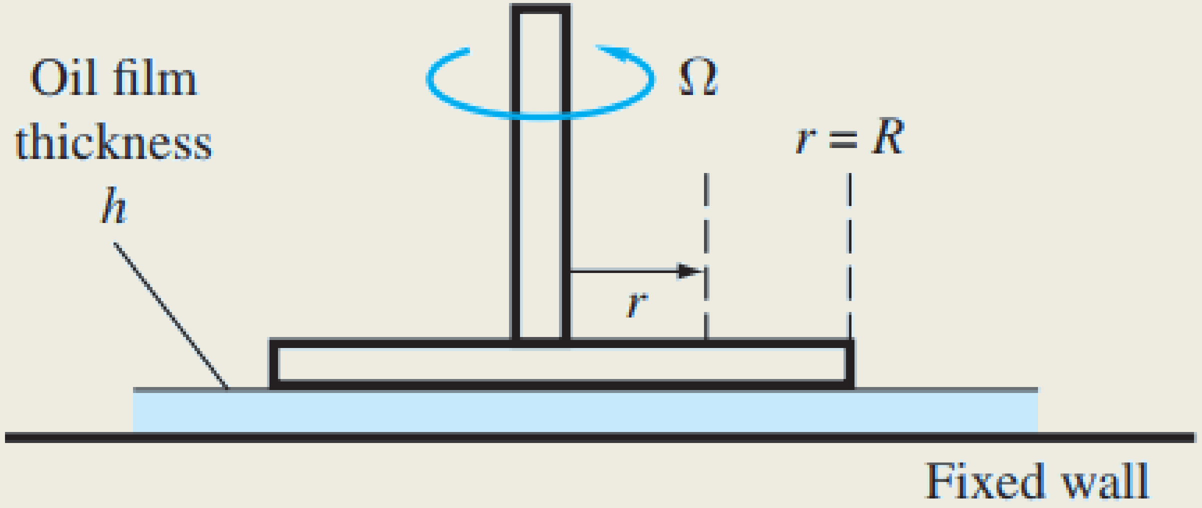
Example

- A viscometer is constructed with two 30-cm-long concentric cylinders, one 20.0 cm in diameter and the other 20.2 cm in diameter. A torque of 0.13 Nm is required to rotate the inner cylinder at 400 rpm (revolutions per minute). Calculate the viscosity.
  - *Step1: conversion of units*
    - $R = d_1/2 = 0.2/2 = 0.1$ m, $h = (d_2\text{-}d_1)/2 = 0.1$ cm = 0.001 m
    - $\omega = 400$ rpm $= 400 * 2\pi/60 = 41.89$ rad/s
  - *Step2:*

$$\mu = \frac{Th}{2\pi R^3 \omega L} = \frac{0.13\ N.m\ (0.001\ m)}{2\pi(0.1\ m)^3\left(41.89\ \frac{rad}{s}\right)(0.3\ m)} = 0.001646\ N.s/m^2$$

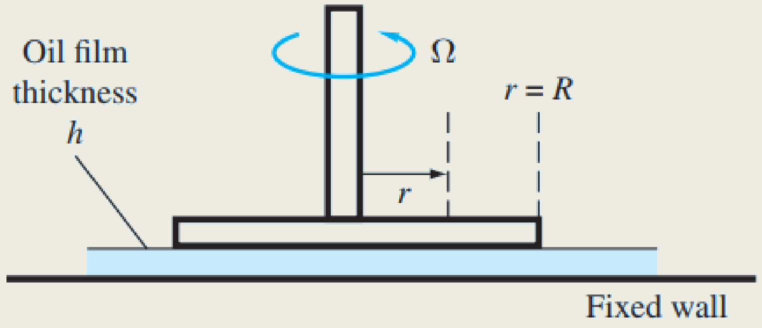
Parallel Plate Viscometer

- Liquid film of viscosity μ and thickness $h << R$ lies between a solid wall and a circular disk rotating at Ω. Derive a formula for the torque M required to rotate the disk.



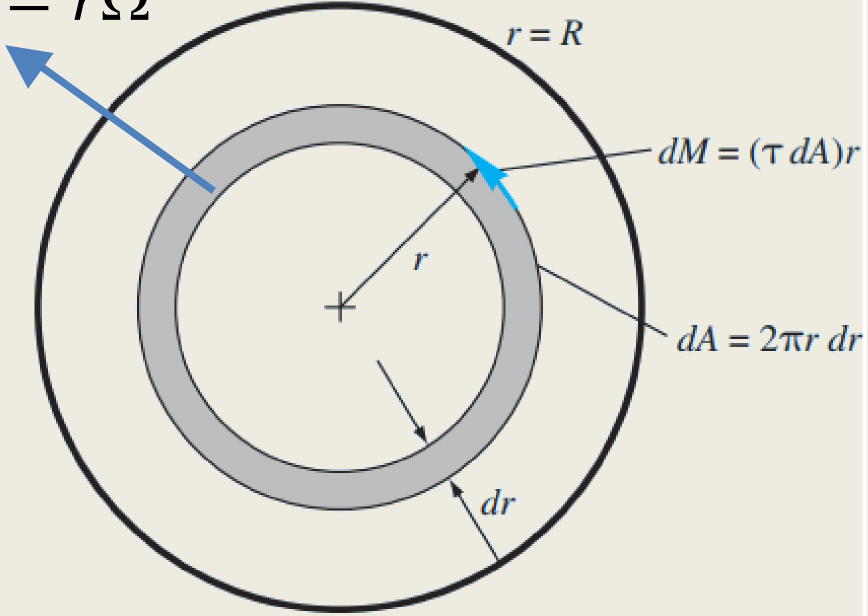
Parallel Plate Viscometer

- **Assumptions:** steady flow, linear velocity, no-slip on the plates, neglecting end effects, air drag on disk



$$u = V\frac{y}{h} \qquad \tau(r) = \mu\frac{V(r)}{h}$$

$$V(r) = r\Omega$$



$$dM = (\tau)(dA)r = \left(\frac{\mu\Omega r}{h}\right)(2\pi r\,dr)r$$

$$M = \int dM = \frac{2\pi\mu\Omega}{h}\int_0^R r^3 dr = \frac{\pi\mu\Omega R^4}{2h}$$