MECHANICS OF FLUIDS

Lecture 2 – Fluid Statics 1

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Note

- All the art-work contents of this lecture are obtained from the following sources, unless otherwise stated:
  - *Fluid Mechanics, 8th edition, Frank M. White, McGraw-Hill, 2016.*
  - *Fluid Mechanics: Fundamental and Applications, 3rd edition, Yunus A. Cengel, John M. Cimbala, McGraw-Hill, 2014.*

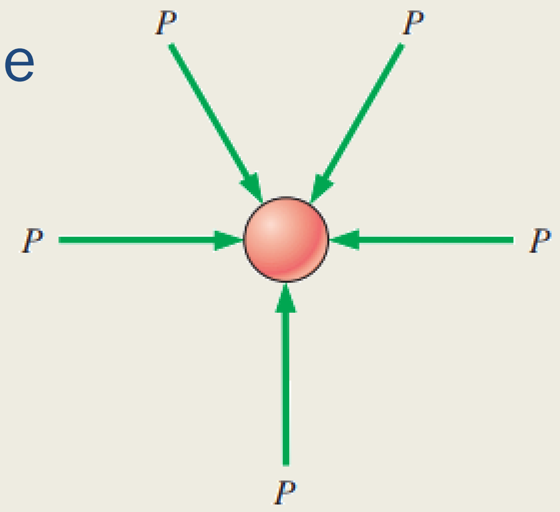
Topics we cover in this lecture

- Some concepts of pressure
- Equation of pressure variation in fluids at rest
- Pressure variations in liquid and gases at rest
- Manometers

source: https://en.wikipedia.org/wiki/Pressure_measurement
source: https://www.oao-7.xyz/

What is pressure?

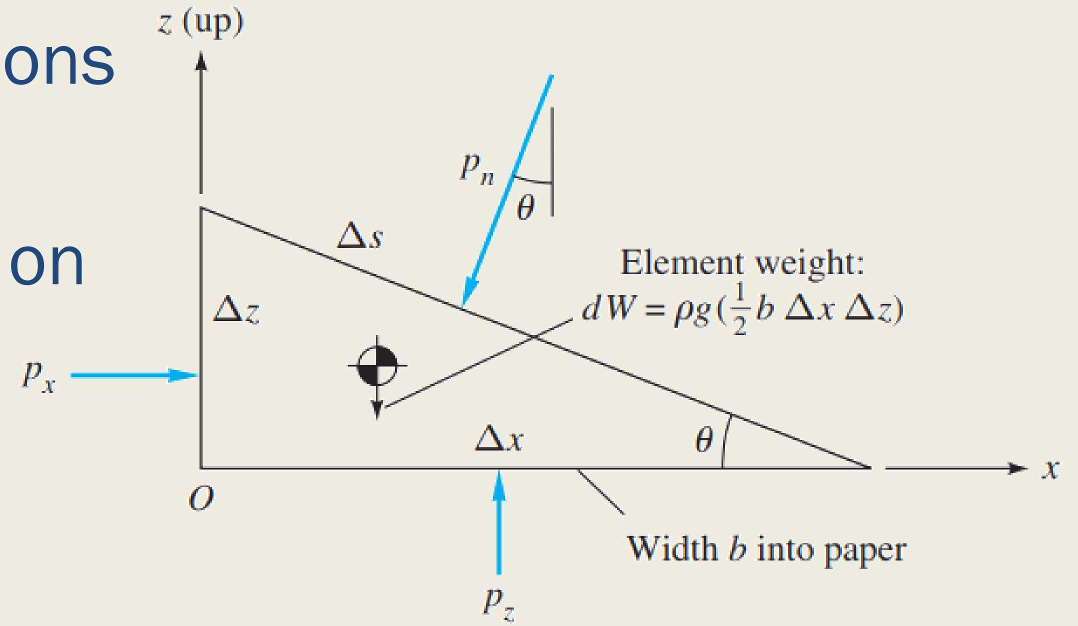
- Pressure is a scalar (thermodynamic) property (like temperature), it is not force and has no direction.
- It exerts normal force/stress on any submerged surface in the fluid.



- It creates compressive force due to molecular contacts (bombardments) on the surface.
- In a fluid at rest, there is no horizontal change in the pressure and the vertical change in the pressure is proportional to density.
  - *How?*

What is pressure? (pressure at a point)

- Consider a **small** wedge with dimensions Δz, Δx and Δs and depth b.
- Horizontal and vertical force balance on this element:



$$\sum F_x = 0 = p_x b\, \Delta z - p_n b\, \Delta s \sin\theta$$

$$\sum F_z = 0 = p_z b\, \Delta x - p_n b\, \Delta s \cos\theta - \tfrac{1}{2}\rho g b\, \Delta x\, \Delta z$$

$$\Delta s \sin\theta = \Delta z$$

$$\Delta s \cos\theta = \Delta x$$

Substitution →

$$p_x = p_n \qquad p_z = p_n + \tfrac{1}{2}\rho g\, \Delta z$$

What is pressure? (pressure at a point)

- If $\Delta z \rightarrow 0$, wedge will become a point and hence:

$$p_x = p_n$$

$$p_z = p_n + \tfrac{1}{2}\rho g \, \Delta z$$

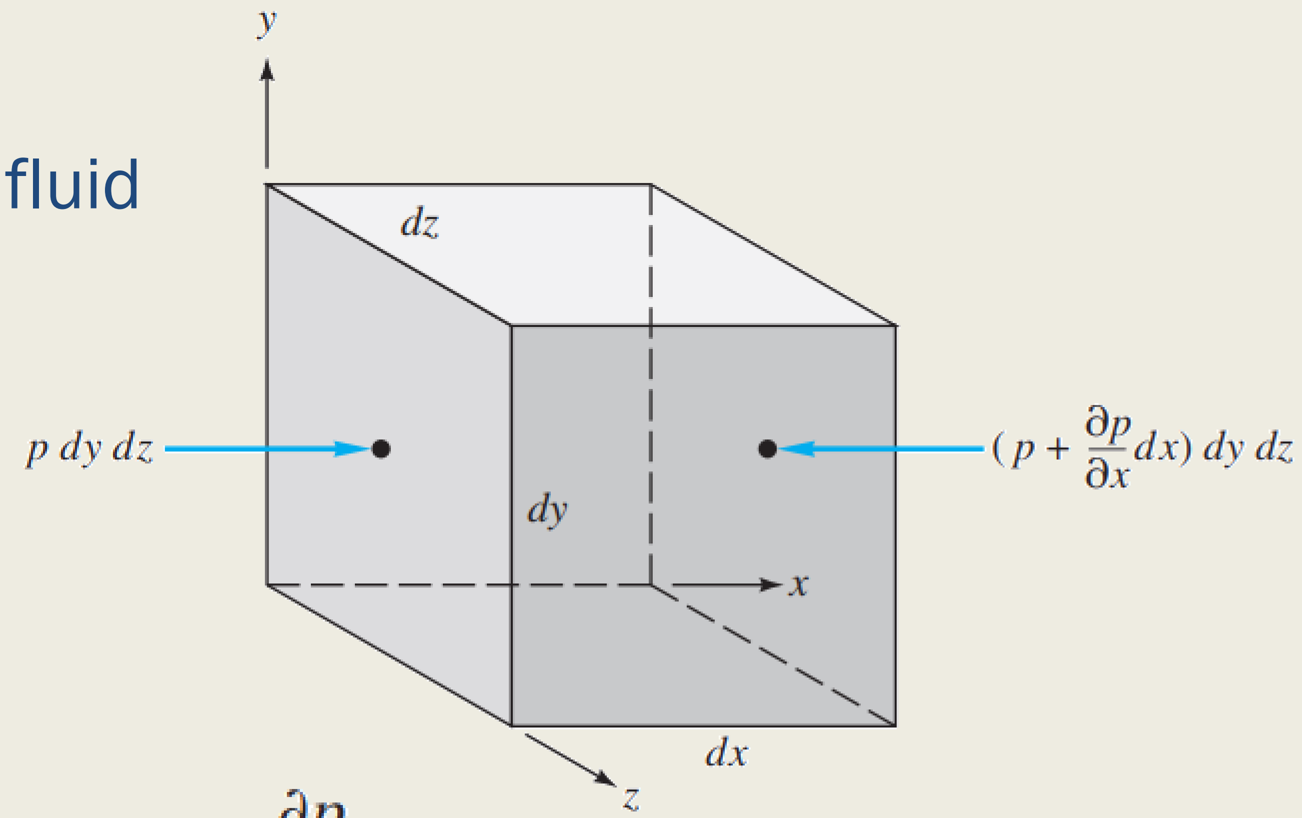
$\Delta z \rightarrow 0$

$$p_x = p_z = p_n = p$$

Pressure in a fluid at rest is a point property and independent of orientation

Pressure variation in fluids

- Force balance on a Cartesian fluid element:



$$dF_x = p\,dy\,dz - \left(p + \frac{\partial p}{\partial x}dx\right)dy\,dz = -\frac{\partial p}{\partial x}dx\,dy\,dz$$

In the same manner we have

$$dF_y = -\frac{\partial p}{\partial y}dx\,dy\,dz \qquad dF_z = -\frac{\partial p}{\partial z}dx\,dy\,dz$$

Pressure variation in fluids

- The net pressure force on the element

$$dF_{press} = \left(-\mathbf{i}\frac{\partial p}{\partial x} - \mathbf{j}\frac{\partial p}{\partial y} - \mathbf{k}\frac{\partial p}{\partial z}\right) dx\, dy\, dz$$

Recall gradient operator

$$\nabla = \text{gradient operator} = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

- The force per unit volume is then:

$\div\, dx\, dy\, dz$ →

$$\mathbf{f}_{press} = -\nabla p$$

- Pressure gradient is the net **surface force** that acts on the sides of the elements.

Pressure variation in fluids

- Gravity is a **body force** acting on the entire mass of the element:

$$d\mathbf{F}_{\text{grav}} = \rho \mathbf{g}\, dx\, dy\, dz \xrightarrow{\div\, dx\, dy\, dz} \mathbf{f}_{\text{grav}} = \rho \mathbf{g}$$

- If the fluid is in the motion, a net surface force due to viscous stresses (will be covered in next lectures in detail) will act on the element which is called $\boldsymbol{f}_{visc}$

Pressure variation in fluids

- Conservation of the linear momentum, Newton's second law

$$\sum \mathbf{f} = \mathbf{f}_{\text{press}} + \mathbf{f}_{\text{grav}} + \mathbf{f}_{\text{visc}} = -\nabla p + \rho \mathbf{g} + \mathbf{f}_{\text{visc}} = \rho \mathbf{a}$$

Linear Momentum (force balance) equation
on the fluid element (close system)

Pressure distribution in fluid at rest

- If the fluid is at rest, the acceleration is zero **a** = 0 and there is no viscous stress/force on the element $\boldsymbol{f}_{visc} = 0$

$$\nabla p = \rho \mathbf{g}$$

- Local gravity when the positive z-direction is considered upward

$$\mathbf{g} = -g\mathbf{k} \qquad \frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\rho g = -\gamma$$

$$\frac{dp}{dz} = -\gamma$$

$$p_2 - p_1 = -\int_1^2 \gamma \, dz$$

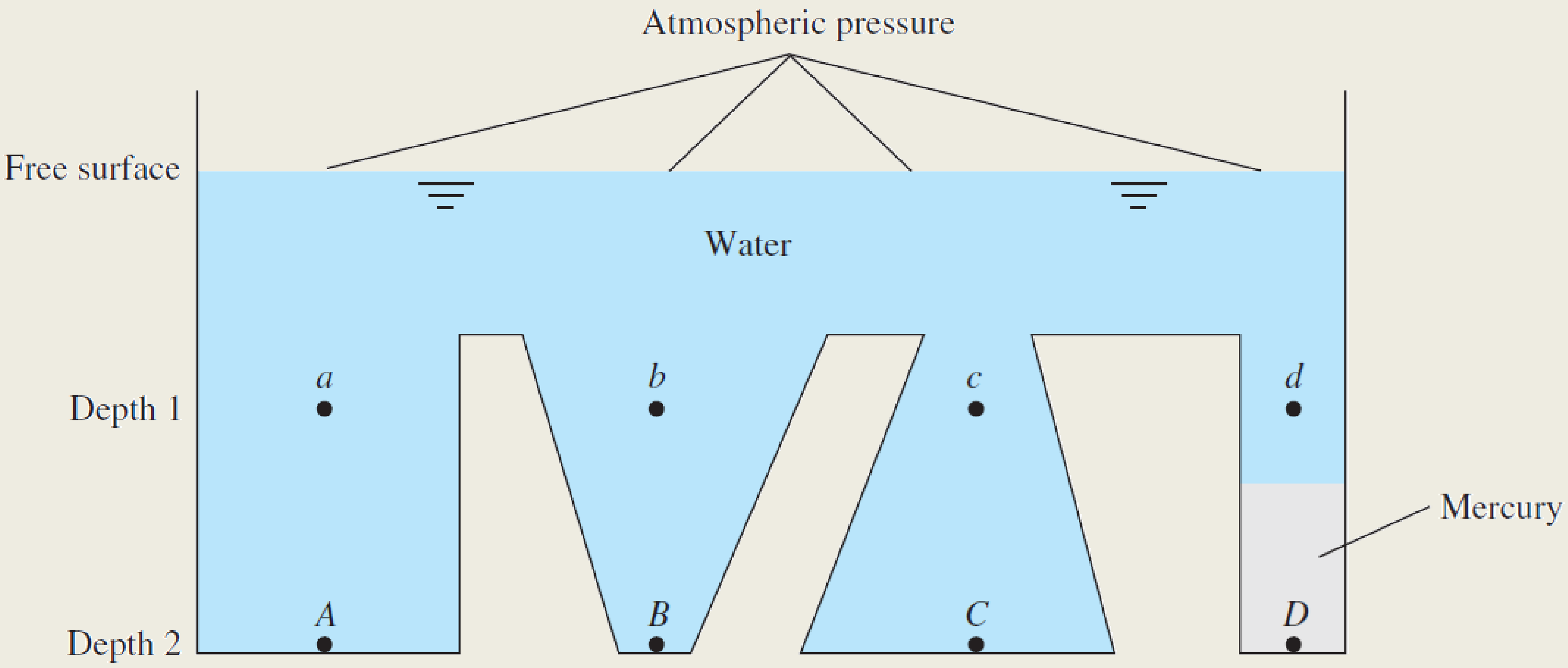
Pressure variation in **liquids** at rest

- For most of liquids at constant temperature, we can neglect the variation of liquid density with pressure (incompressible).

$$p_2 - p_1 = -\int_1^2 \gamma \, dz \quad \xrightarrow{\rho = \text{cte}} \quad p_2 - p_1 = -\gamma\,(z_2 - z_1)$$

Fluid	Specific weight γ at 68°F = 20°C lbf/ft^3	N/m^3
Air (at 1 atm)	0.0752	11.8
Ethyl alcohol	49.2	7,733
SAE 30 oil	55.5	8,720
Water	62.4	9,790
Seawater	64.0	10,050
Glycerin	78.7	12,360
Carbon tetrachloride	99.1	15,570
Mercury	846	133,100

Pressure variation in **liquids** at rest



1) Pressures at points a, b, c, and d are equal

2) Pressures at points A, B, C are equal

3) Pressure at D is greater than C

Pressure variation in gases at rest (in atmosphere)

- Gases are compressible, and density changes with pressure. Assuming ideal gas law, $p = \rho RT$

$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT} g \quad \Rightarrow \quad \int_1^2 \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_1^2 \frac{dz}{T}$$

- Assuming constant temperature for the atmosphere, T_0:

$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$ Eq. (1)

Pressure variation in gases at rest (in atmosphere)

- Assuming linear variation of temperature (in troposphere, sea level to 36000 ft):

$$T \approx T_0 - Bz$$

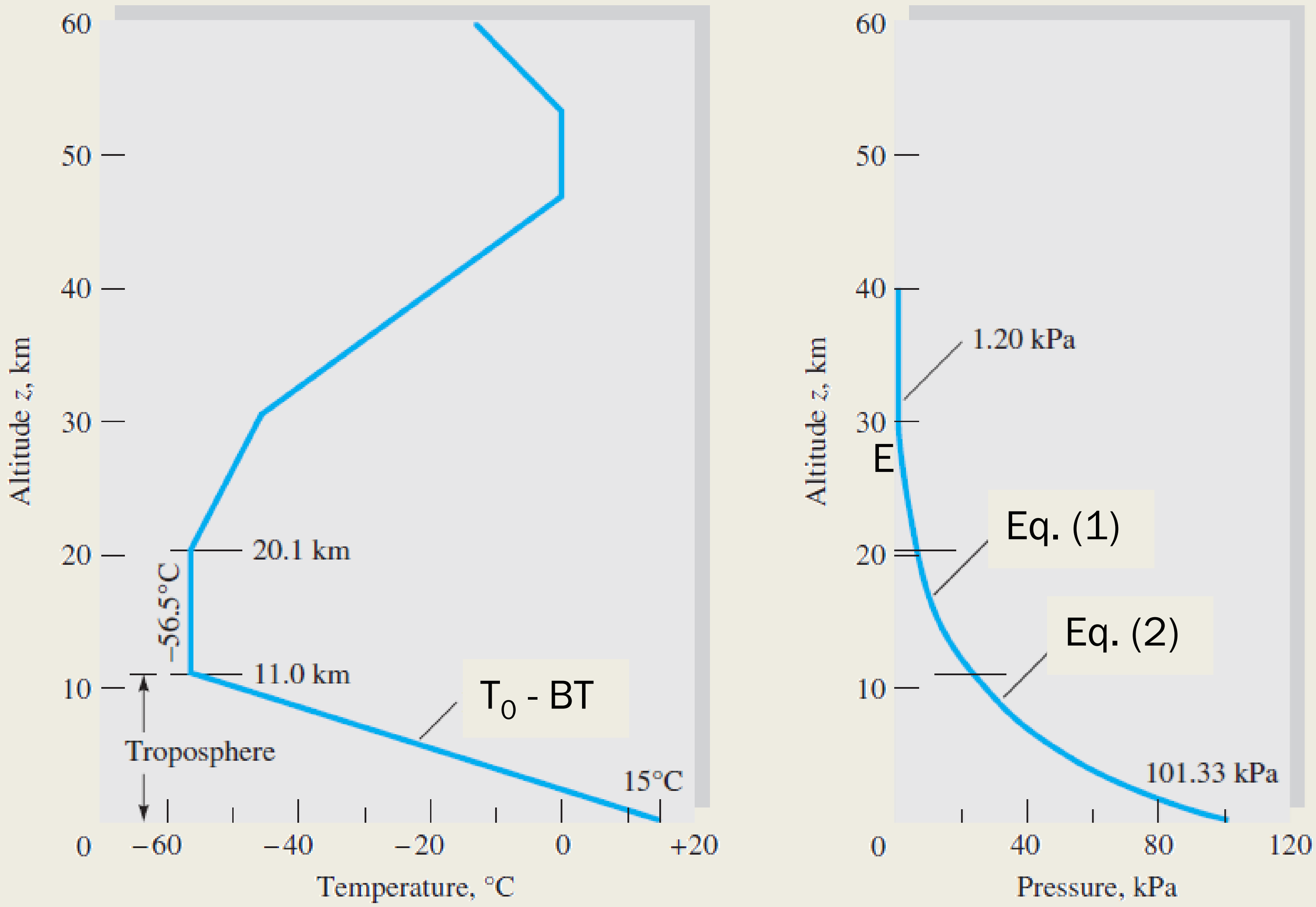
With
T_0 = 288.15 K
B = 0.0065 K/m

- Pressure variation equation in troposphere (sea level to 36000 ft):

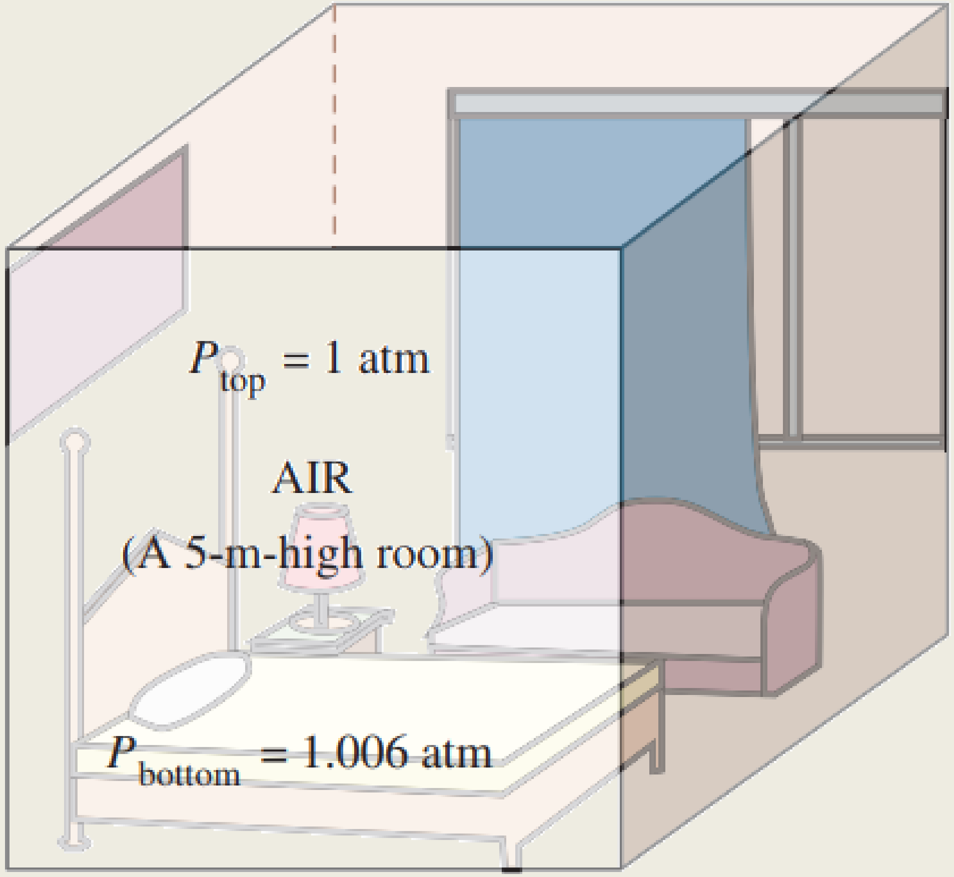
$$p = p_a \left(1 - \frac{Bz}{T_0}\right)^{g/(RB)} \quad \text{where} \quad \frac{g}{RB} = 5.26 \text{ (air)}$$

Eq. (2)

Pressure variation in gases at rest (in atmosphere)



Pressure variation in gases at rest (short distances)



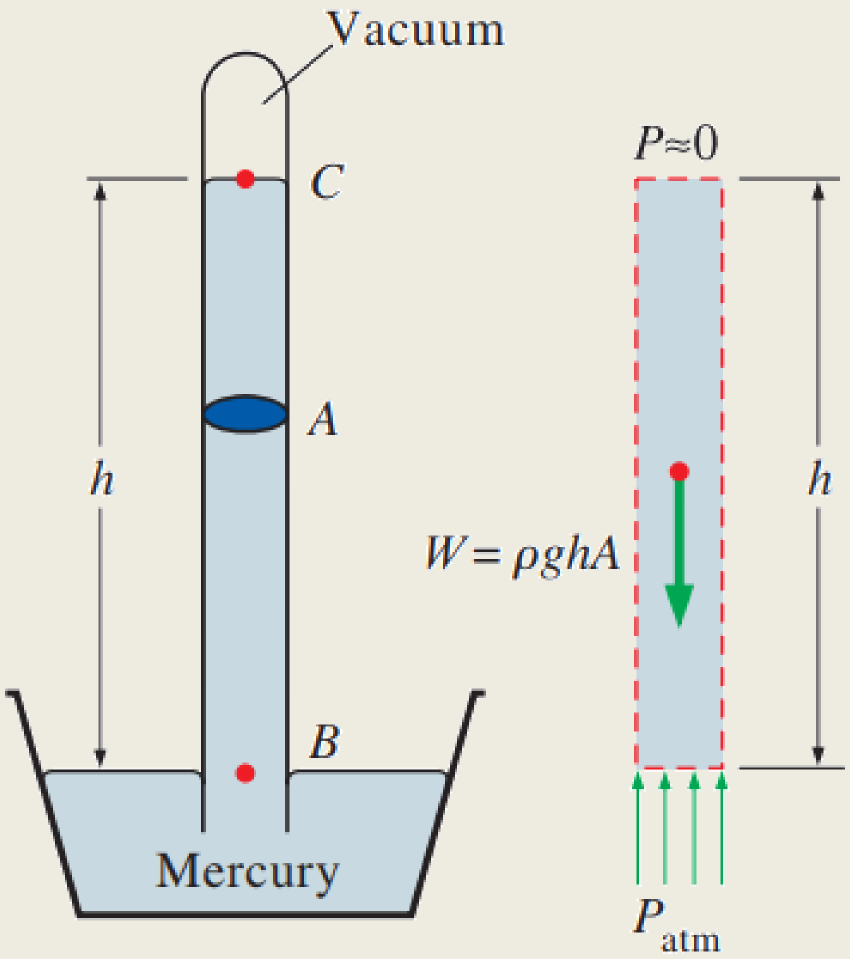
Barometer

- It is used to measure the atmospheric pressure.
- Evangelista Torricelli (16th century)

$$P_{atm} = \rho g h$$

15595 kg/m^3 | 9.806 m/s^2 | Height of mercury column above point B

At standard atmosphere h reads 760 mm.



Example

- If sea-level pressure is 101,350 Pa, compute the standard pressure at an altitude of 5000 m, using (*a*) the exact formula and (*b*) an isothermal assumption at a standard sea-level temperature of 15 °C. Is the isothermal approximation adequate?

Answer:

Part (a): 54000 Pa = 54 kPa

Part (b): 56000 Pa = 56 kPa

The difference is around 4%.

It can be shown that this error is less than 1% for elevation variations up to 200 m

Example

- Part (a):

$$p = p_a\left(1 - \frac{Bz}{T_0}\right)^{g/(RB)}$$

With
T_0 = 288.15 K
B = 0.0065 K/m

R = 8314 J/(K.kmol)
For air: 8314/29 = 287 J/(K.kg)
= 287 $m^2/(K.s^2)$

$$p = p_a\left[1 - \frac{(0.00650\ \text{K/m})(5000\ \text{m})}{288.16\ \text{K}}\right]^{5.26} = (101{,}350\ \text{Pa})(0.8872)^{5.26}$$

$$= 101{,}350(0.5328) = 54{,}000\ \text{Pa}$$

- Part (b):

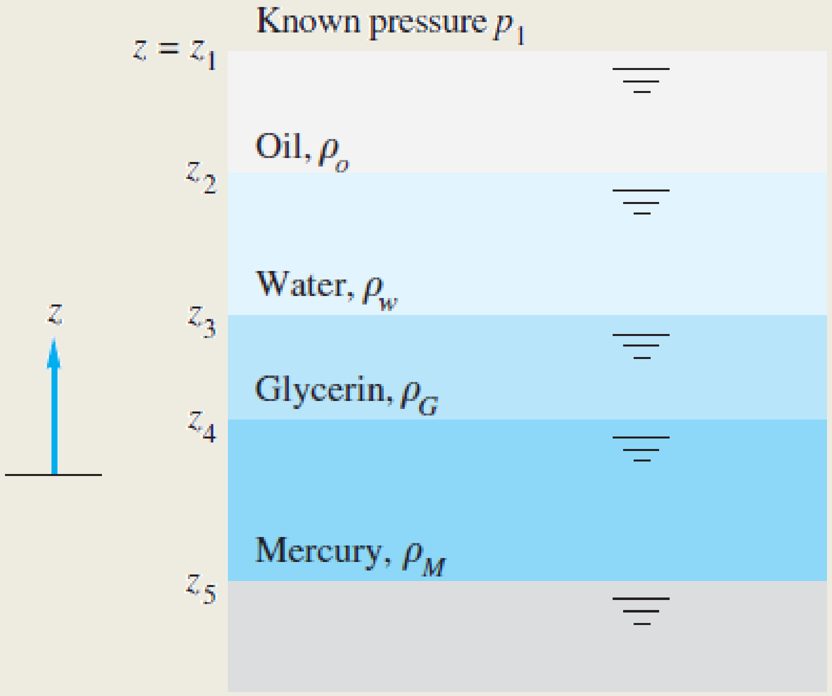
$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

$$p \approx p_a \exp\left(-\frac{gz}{RT}\right) = (101{,}350\ \text{Pa}) \exp\left\{-\frac{(9.807\ \text{m/s}^2)(5000\ \text{m})}{[287\ \text{m}^2/(\text{s}^2 \cdot \text{K})](288.16\ \text{K})}\right\}$$

$$= (101{,}350\ \text{Pa}) \exp(-0.5929) \approx 56{,}000\ \text{Pa}$$

Multiple layers of fluid

- How does the pressure changes into the depth of a liquid column?



recall: $p_2 - p_1 = -\gamma\,(z_2 - z_1)$

$$p_2 - p_1 = -\rho_o g(z_2 - z_1)$$

$$p_3 - p_2 = -\rho_w g(z_3 - z_2)$$

$$p_4 - p_3 = -\rho_G g(z_4 - z_3)$$

$$p_5 - p_4 = -\rho_M g(z_5 - z_4)$$

$$\text{Sum} = p_5 - p_1$$

$$p_5 - p_1 = -\gamma_0(z_2 - z_1) - \gamma_w(z_3 - z_2) - \gamma_G(z_4 - z_3) - \gamma_M(z_5 - z_4)$$

$$p_5 = p_1 + \gamma_0\,|z_1 - z_2| + \gamma_w\,|z_2 - z_3| + \gamma_G\,|z_3 - z_4| + \gamma_M\,|z_4 - z_5|$$

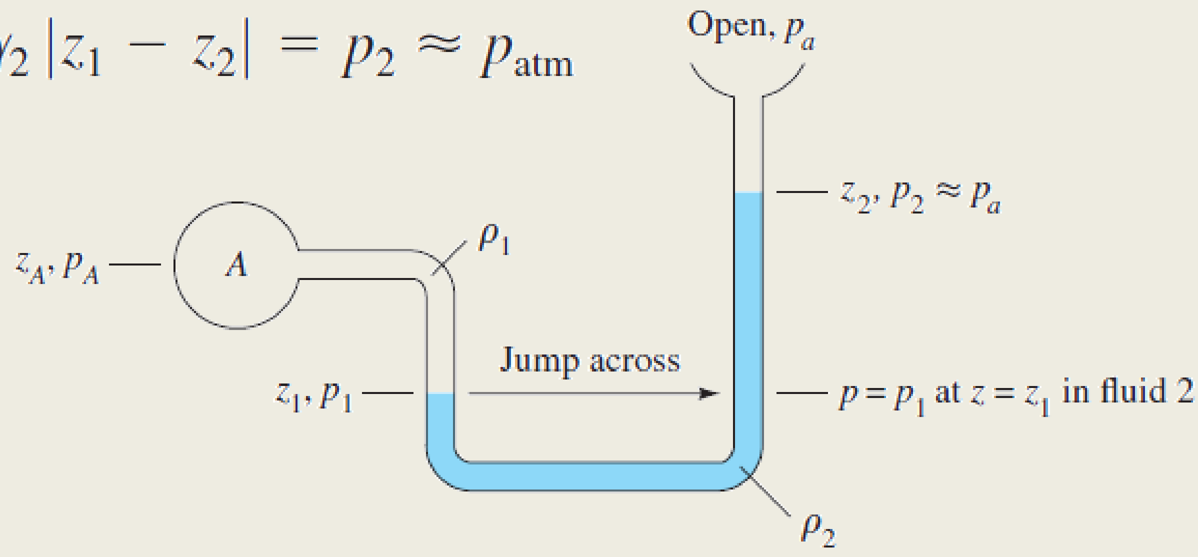
Multiple layers of fluid

A simple rule: Pressure increase downward and pressure decrease upward in the fluid column

Manometer

- A **manometer** is an open U-tube device which measures the **gage** of a point relative to the atmosphere.

$$p_A + \gamma_1 |z_A - z_1| - \gamma_2 |z_1 - z_2| = p_2 \approx p_{atm}$$



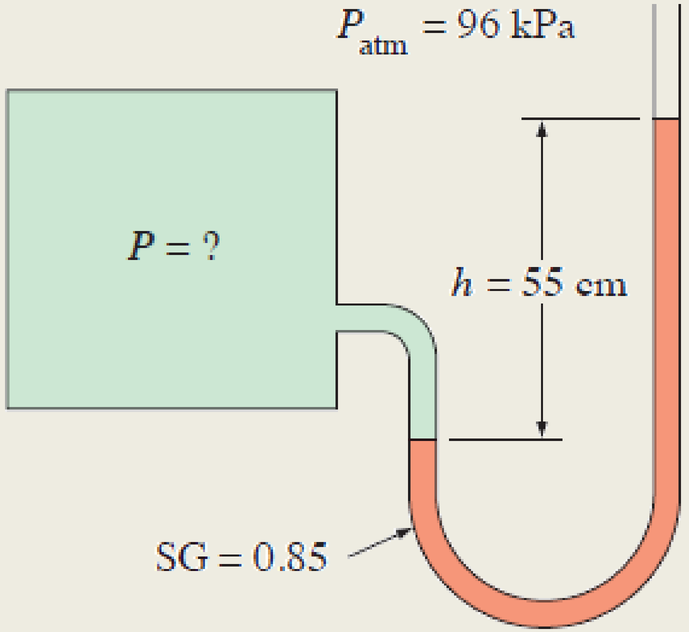
1) If fluid 1 (A) is the air, ρ_1 can be ignored and the height difference reading is the gage pressure.
2) For a better resolution (or for low pressure differences) the water or oil is selected and for high pressure changes mercury.

Example 2

- A manometer is used to measure the pressure of a gas in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.

P_{tank} = 96 kPa + 850 kg/m^3 × 9.81 m/s^2 × 0.55 m =
96 kPa + 4586 N/m2 or Pa =
96 kPa + 4.6 kPa = 100.6 kPa

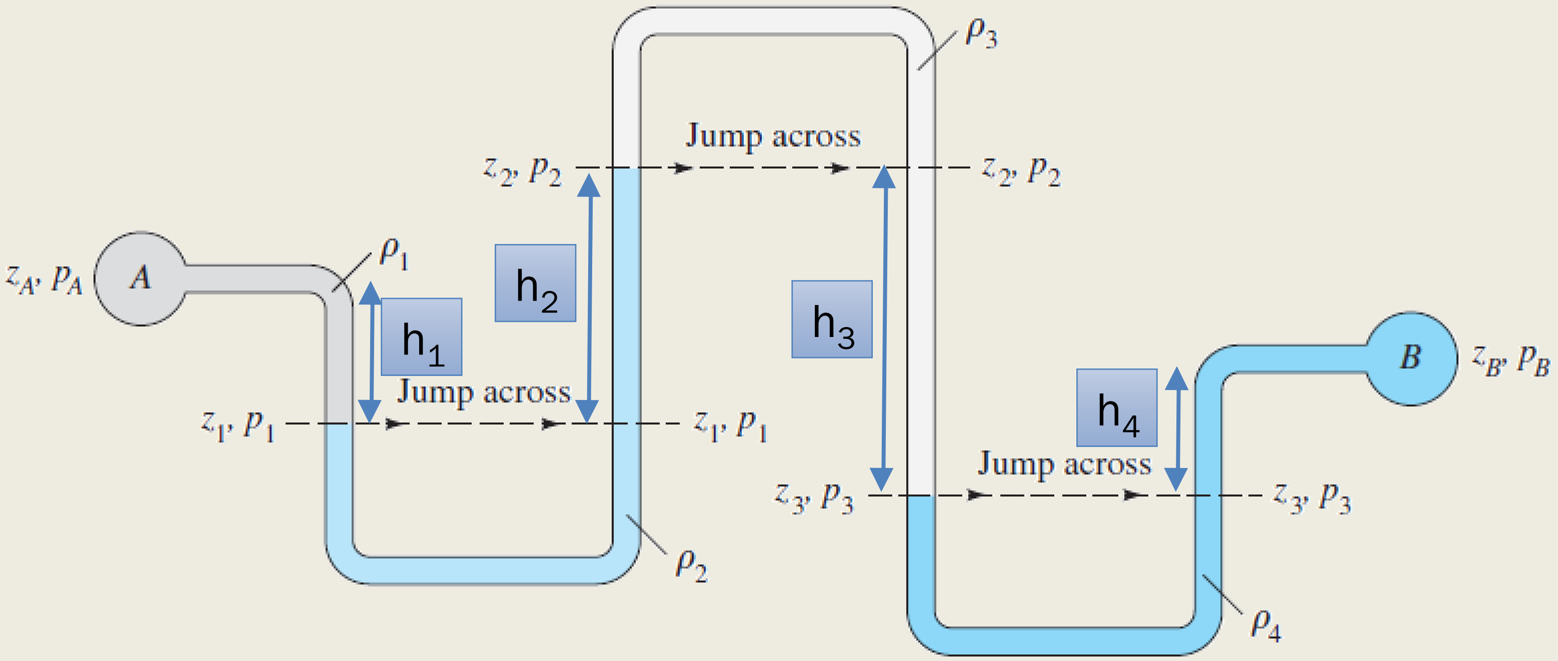
Recall: 1 N = 1 kg.m/s^2



Example 3

- What is the relation between pressures at points A and B?

$$p_A + h_1\gamma_1 - h_2\gamma_2 + h_3\gamma_3 - h_4\,\gamma_4 = p_B$$



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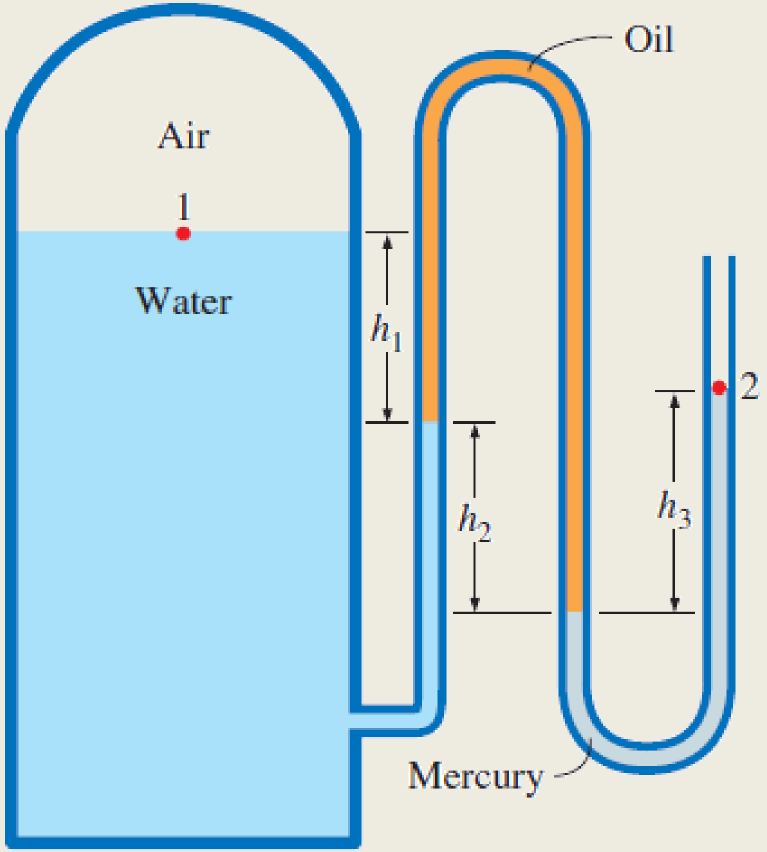
Example 4

- Water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if $h_1 = 0.1$ m, $h_2 = 0.2$ m, and $h_3 = 0.35$ m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.



Example 4

$$P_{Air} + h_1\,\gamma_w + h_2\,\gamma_{oil} - h_3\,\gamma_M = P_2 = P_{atm}$$

$$P_{Air} + 0.1\text{ m }(1000\times 9.81\text{ N/m}^3) +$$
$$0.2\text{ m }(850\times 9.81\text{ N/m}^3) -$$
$$0.35\text{ m }(13600*9.81\text{ N/m}^3) = 85.6\text{ kPa}$$

$$P_{Air} + 981\text{ Pa} + 1667.7\text{ Pa} - 46696\text{ Pa} = 85.6\text{ kPa}$$

$$P_{Air} = 85.6\text{ kPa} + (44046/1000)\text{kPa} = 129.7\text{ kPa}$$