MECHANICS OF FLUIDS

Lecture 3 – Fluid Statics 2

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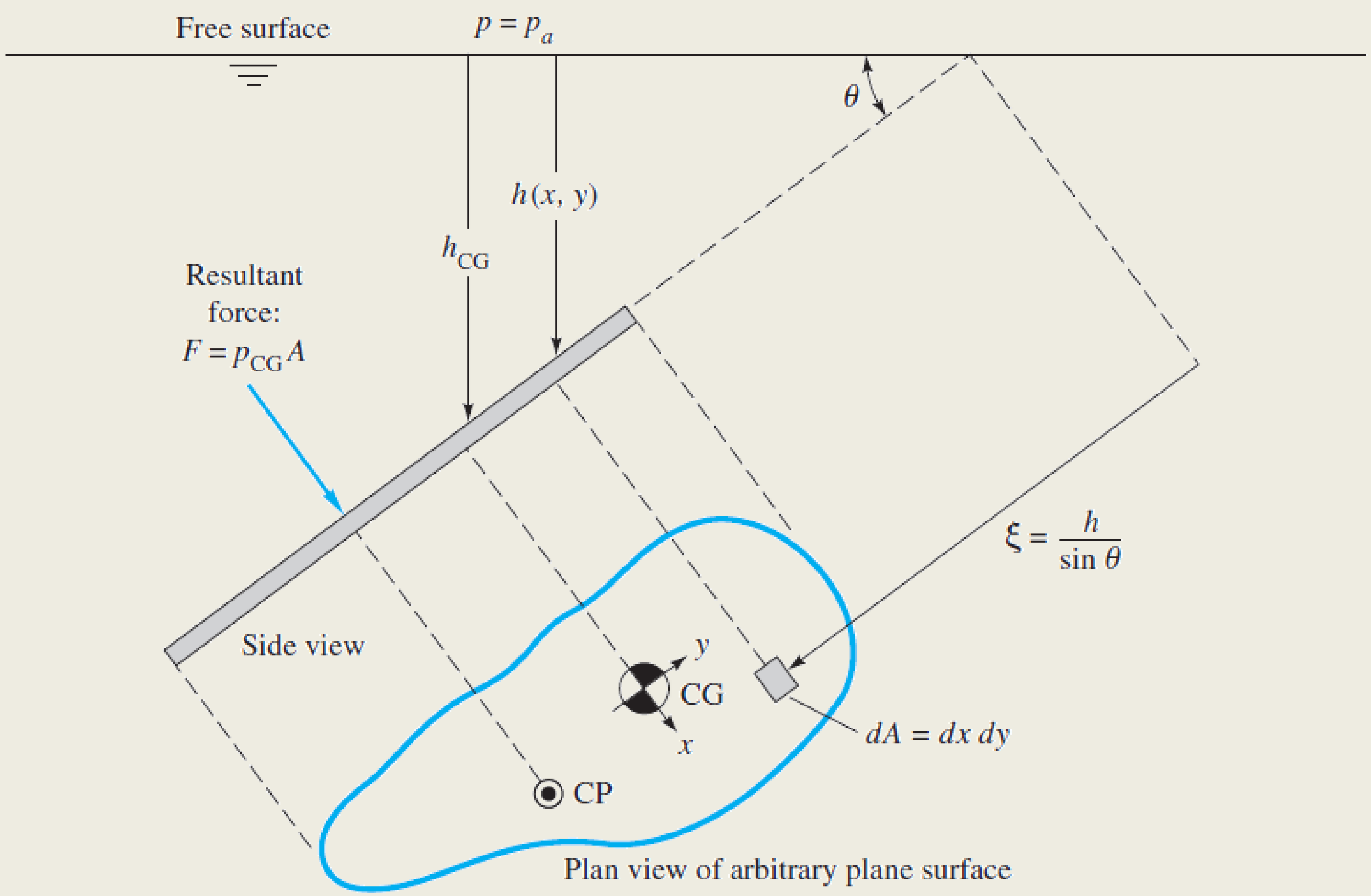
Note

- All the art-work contents of this lecture are obtained from the following sources, unless otherwise stated:
  - *Fluid Mechanics, 8th edition, Frank M. White, McGraw-Hill, 2016.*
  - *Fluid Mechanics: Fundamental and Applications, 3rd edition, Yunus A. Cengel, John M. Cimbala, McGraw-Hill, 2014.*

Pressure force on a plane surface

Pressure on dA

$$p = p_a + \gamma h$$



Pressure force on a plane surface

- Total force on the plane

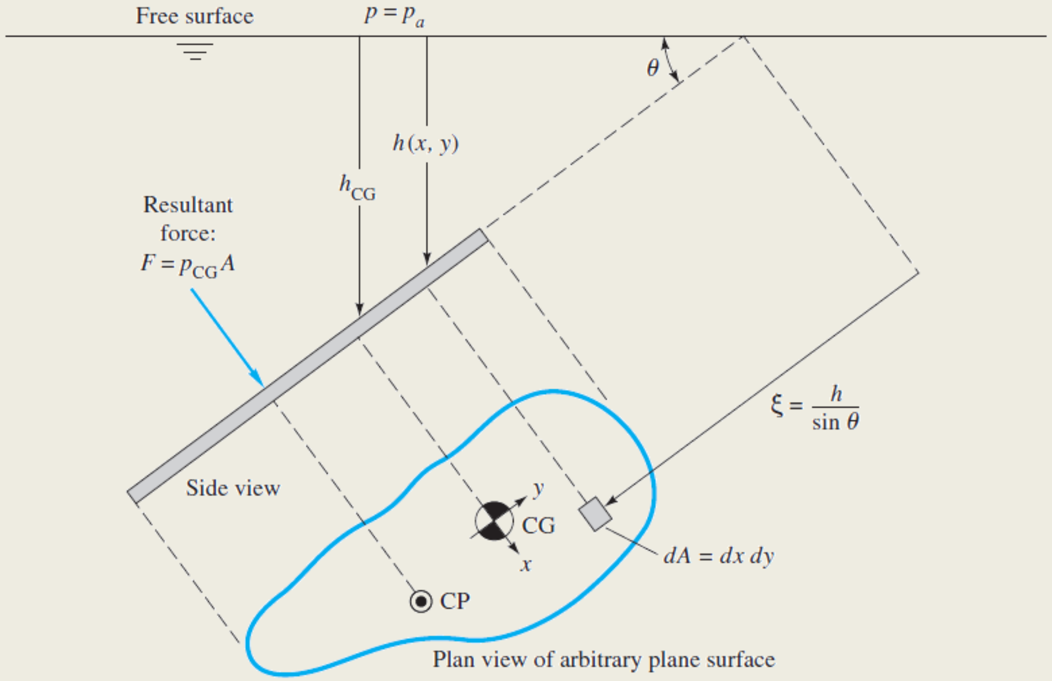
$$F = \int p\, dA = \int (p_a + \gamma h)\, dA = p_a A + \gamma \int h\, dA$$

$$h = \xi \sin \theta$$

$$F = p_a A + \gamma \sin \theta \int \xi\, dA = p_a A + \gamma \sin \theta\, \xi_{CG} A$$

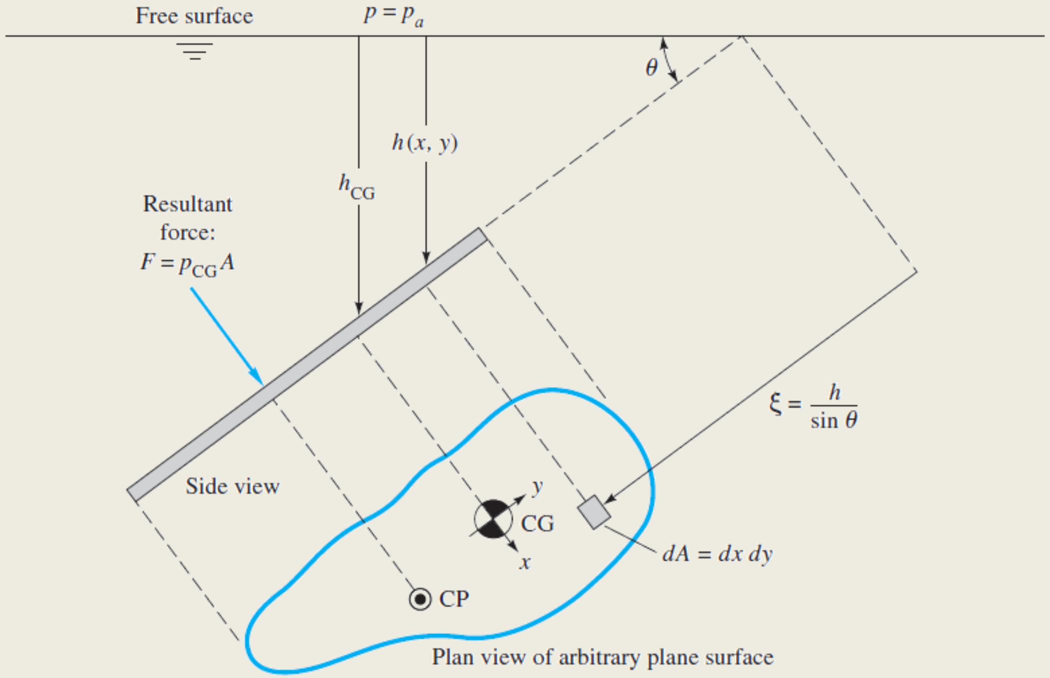
$$\xi_{CG} = \frac{1}{A} \int \xi\, dA$$

$$\xi_{CG} \sin \theta = h_{CG}$$



Pressure force on a plane surface

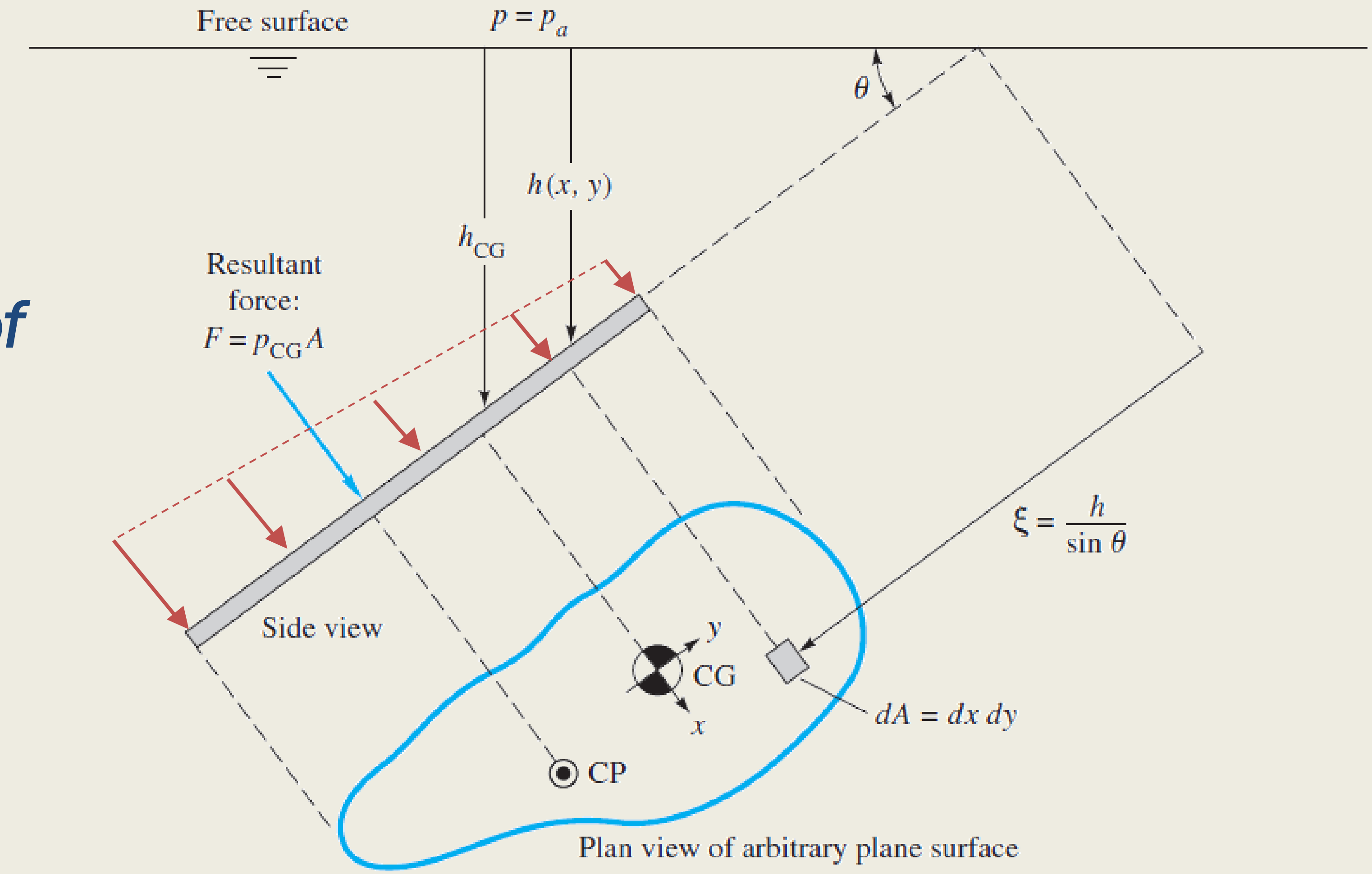
- The force is the sum of **atmospheric pressure force** plus the **column high of the liquid** from surface to the centroid of the plate.



$$F = p_a A + \gamma h_{CG} A = (p_a + \gamma h_{CG})A = p_{CG} A$$

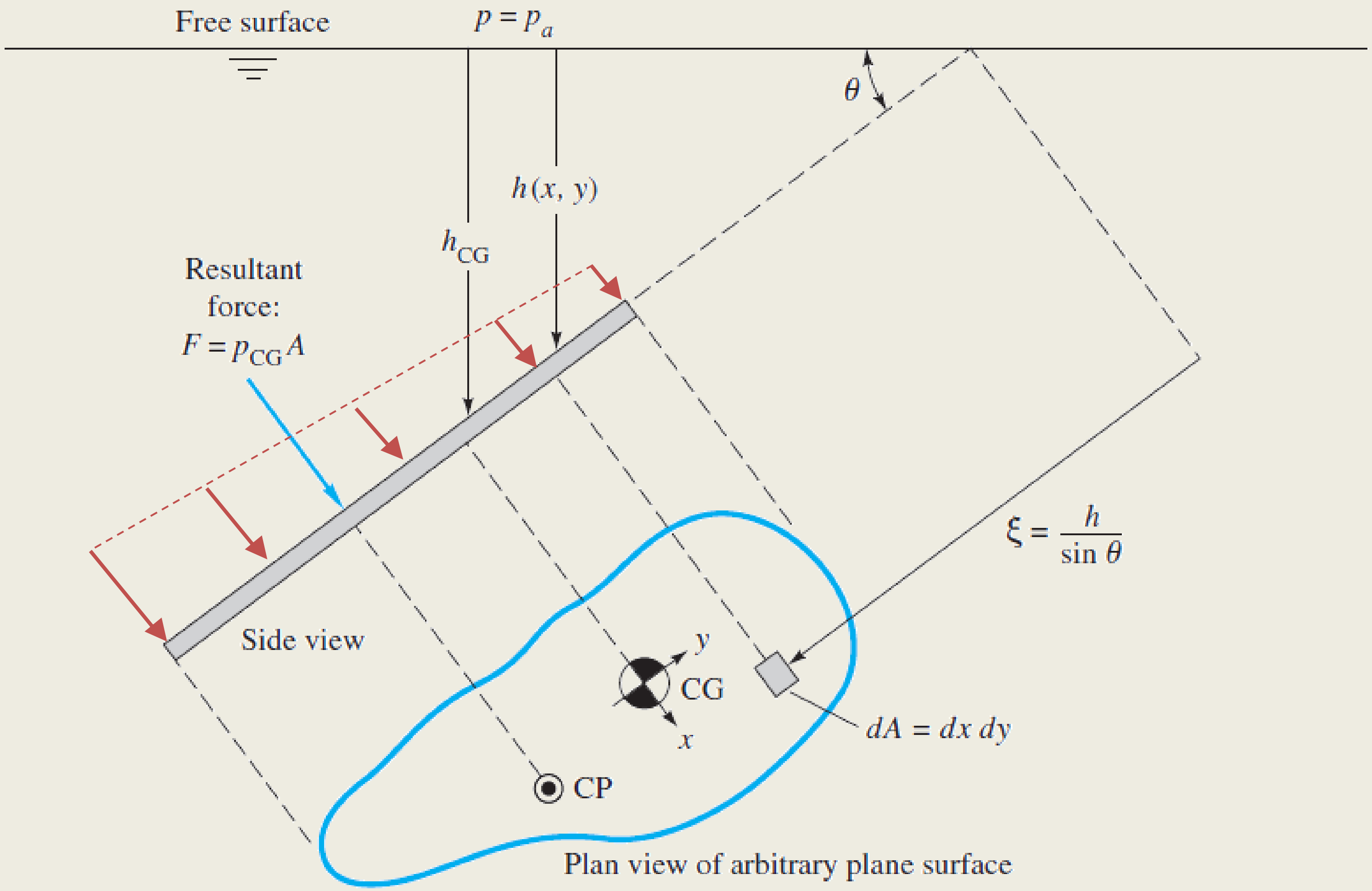
Pressure force on a plane surface

- Center of pressure:
  - *The point at which the total resultant force is acting is called the* ***center of pressure***



Pressure force on a plane surface

- The total force is assumed to be acting on a point where the moment of the **total force around the CG** is equal to the **sum of the moments of distributed pressure forces around the CG.**



Pressure force on a plane surface

$$F y_{CP} = \int yp\, dA = \int y(p_a + \gamma\xi \sin\theta)\, dA = \gamma \sin\theta \int y\xi\, dA$$

$$\int p_a y\, dA = 0$$

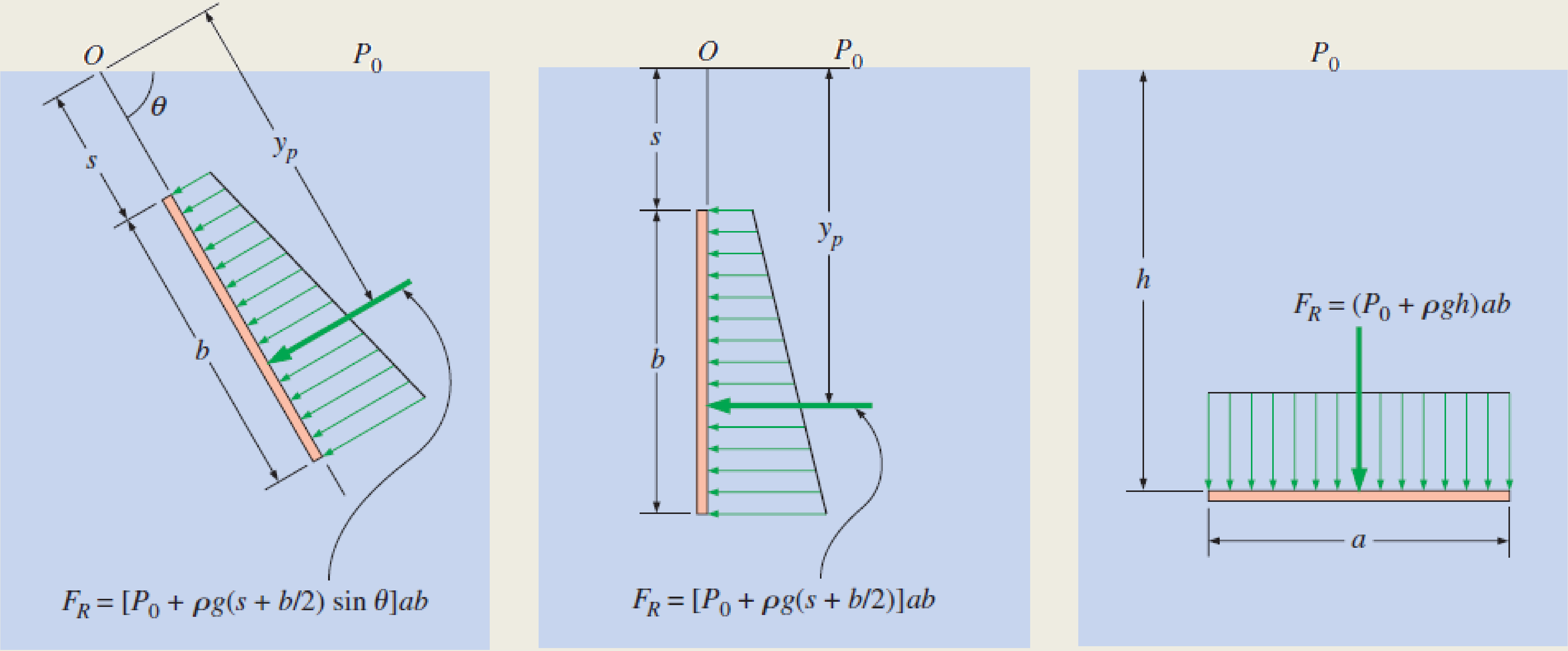
$$\xi = \xi_{CG} - y$$

$$F y_{CP} = \gamma \sin\theta \left(\xi_{CG} \int y\, dA - \int y^2\, dA \right) = -\gamma \sin\theta\, I_{xx}$$

0

$$y_{CP} = -\gamma \sin\theta \frac{I_{xx}}{p_{CG} A}$$

Pressure force on a plane surface



Pressure force on a plane surface

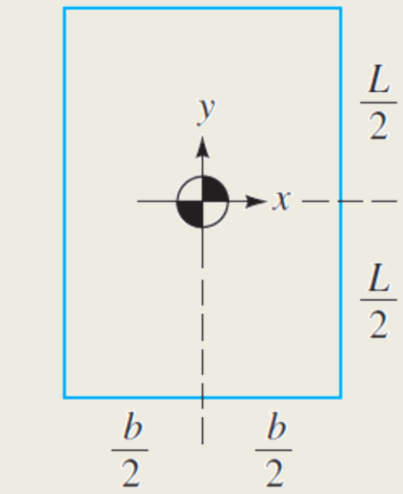
- With a similar approach, we can find the x_{CP} (if there is asymmetry around y axis):

$$F x_{CP} = \int xp\, dA = \int x[p_a + \gamma(\xi_{CG} - y) \sin\theta]\, dA$$

$$= -\gamma \sin\theta \int xy\, dA = -\gamma \sin\theta\, I_{xy}$$

$$x_{CP} = -\gamma \sin\theta \frac{I_{xy}}{p_{CG} A}$$

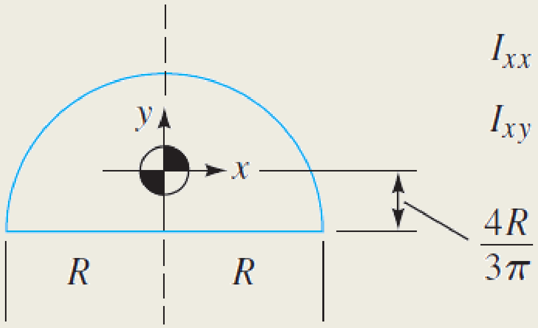
Pressure force on a plane surface



$$A = bL$$

$$I_{xx} = \frac{bL^3}{12}$$

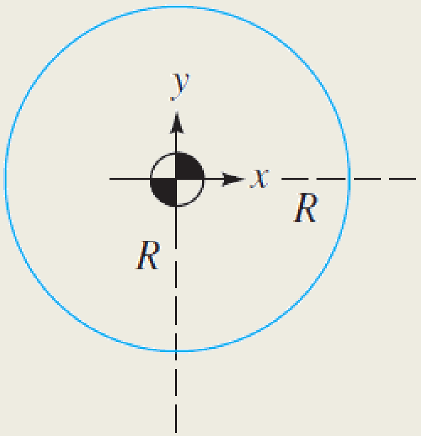
$$I_{xy} = 0$$



$$A = \frac{\pi R^2}{2}$$

$$I_{xx} = 0.10976R^4$$

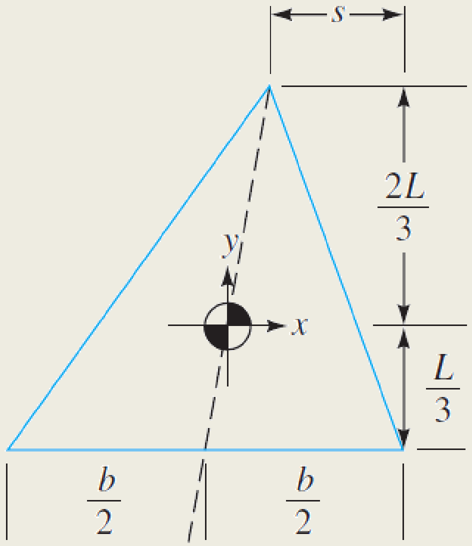
$$I_{xy} = 0$$



$$A = \pi R^2$$

$$I_{xx} = \frac{\pi R^4}{4}$$

$$I_{xy} = 0$$



$$A = \frac{bL}{2}$$

$$I_{xx} = \frac{bL^3}{36}$$

$$I_{xy} = \frac{b(b-2s)L^2}{72}$$

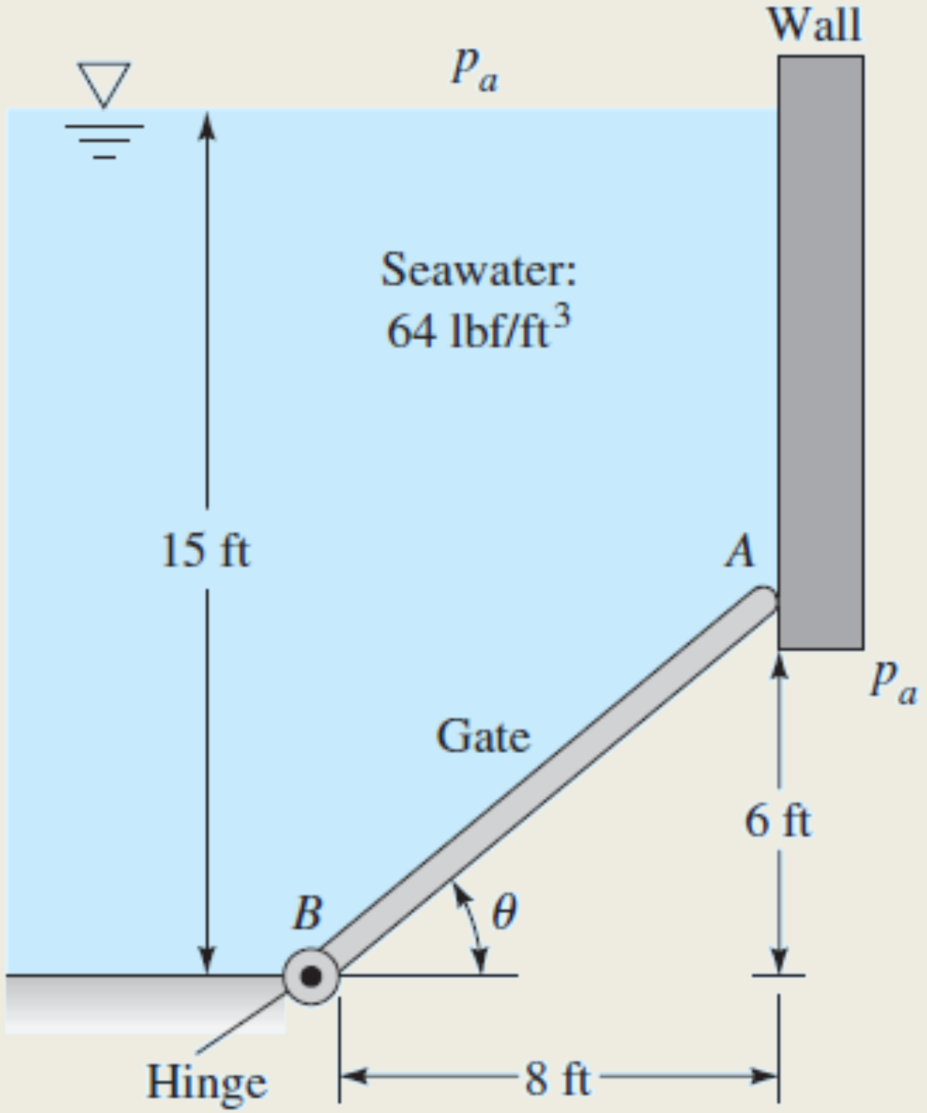
Example 1

The gate shown in figure is 5 ft wide, is hinged at point *B* , and rests against a smooth wall at point *A* . Compute:

1) The force on the gate due to seawater pressure

2) The horizontal force *P* exerted by the wall at point *A*

3) The reactions at the hinge *B*



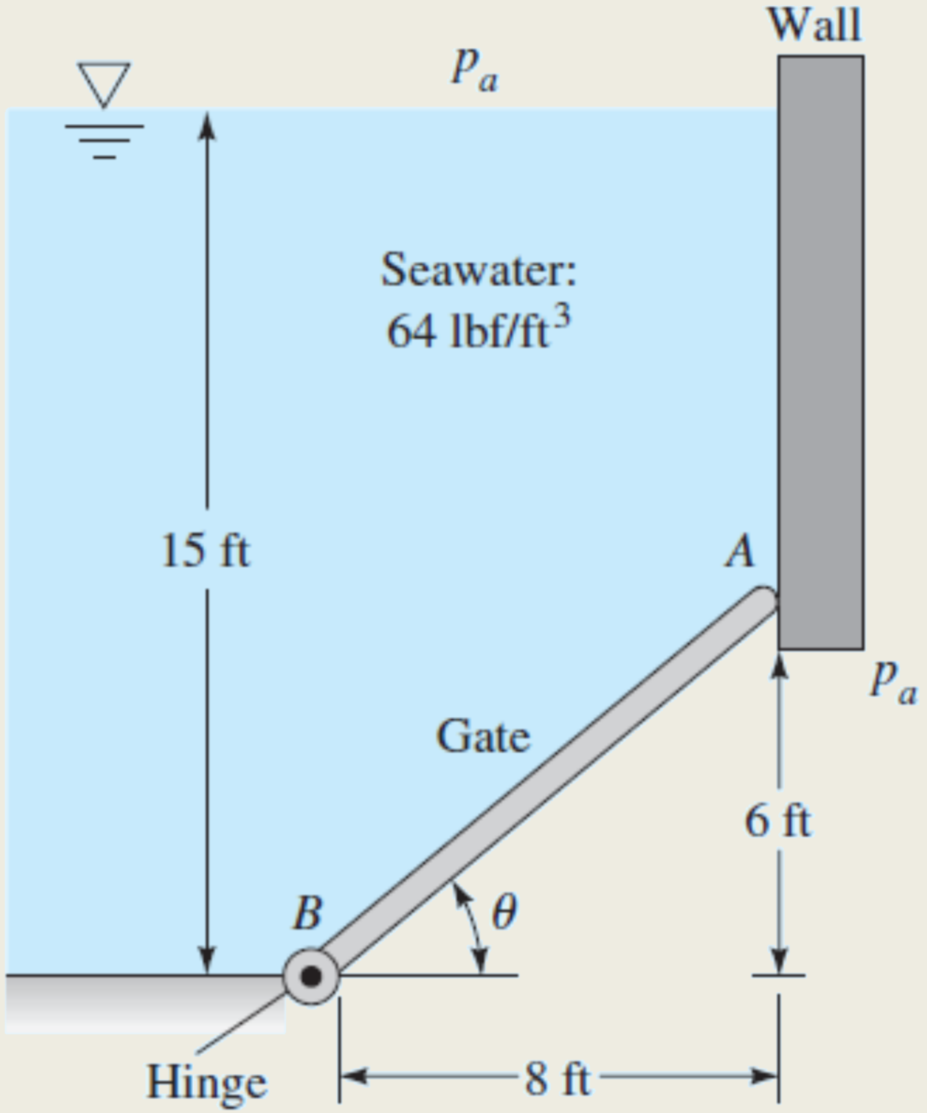
Example 1

- **Part 1:**

Plate area: A = 10×5 = 50 ft^2

h_{CG} = 9 + 3 = 12 ft

F = $\gamma\, h_{CG}$ A = 64 (lbf/ft^3)× 12 ft × 50 ft^2 = 38400 lbf



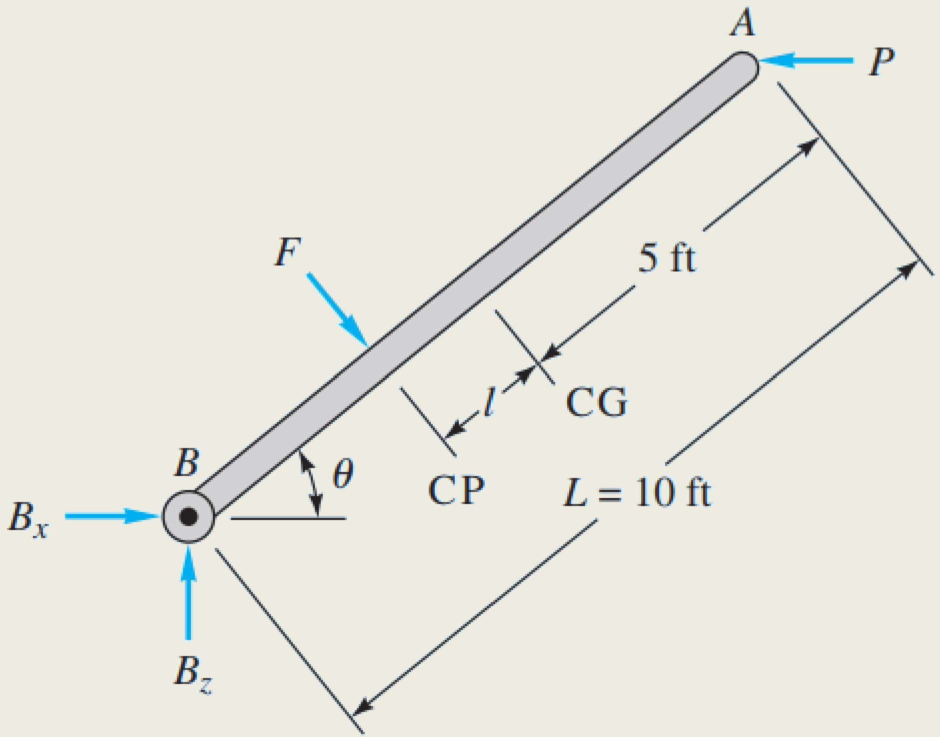
Example 1

- Part 2:
  - *Center of pressure*

$$I_{xy} = 0 \text{ and } I_{xx} = \frac{bL^3}{12} = \frac{(5\text{ ft})(10\text{ ft})^3}{12} = 417\text{ ft}^4$$

$$l = -y_{CP} = +\frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{(417\text{ ft}^4)(\frac{6}{10})}{(12\text{ ft})(50\text{ ft}^2)} = 0.417\text{ ft}$$

  - *Calculating P by balancing moments around B*



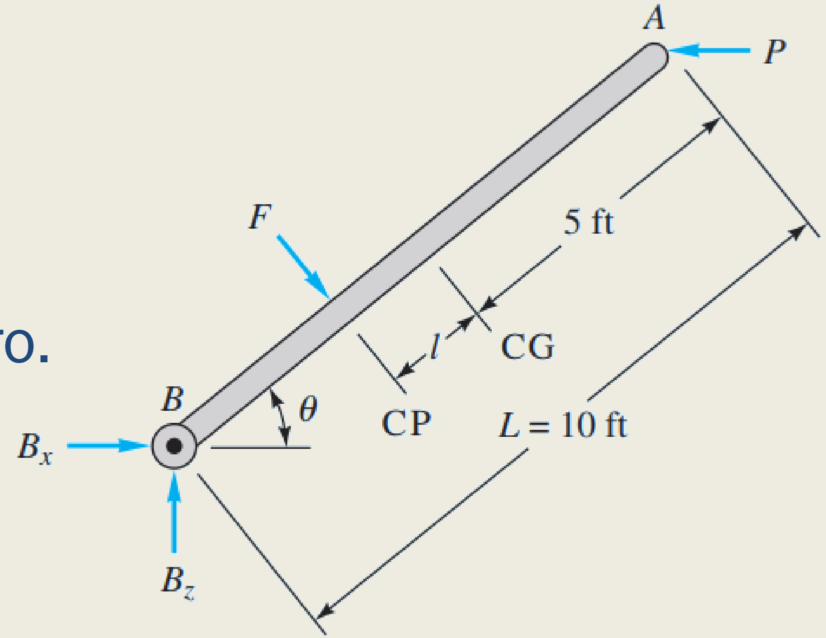
$$PL\sin\theta - F(5 - l) = P(6\text{ ft}) - (38{,}400\text{ lbf})(4.583\text{ ft}) = 0$$

$$P = 29{,}300\text{ lbf}$$

Example 1

- Part 3:

Since the gate is not moving (linearly), Sum of the horizontal and vertical forces on the gate should be zero.



$$\sum F_x = 0 = B_x + F \sin \theta - P = B_x + 38{,}400 \text{ lbf } (0.6) - 29{,}300 \text{ lbf}$$

$$B_x = 6300 \text{ lbf}$$

$$\sum F_z = 0 = B_z - F \cos \theta = B_z - 38{,}400 \text{ lbf } (0.8)$$

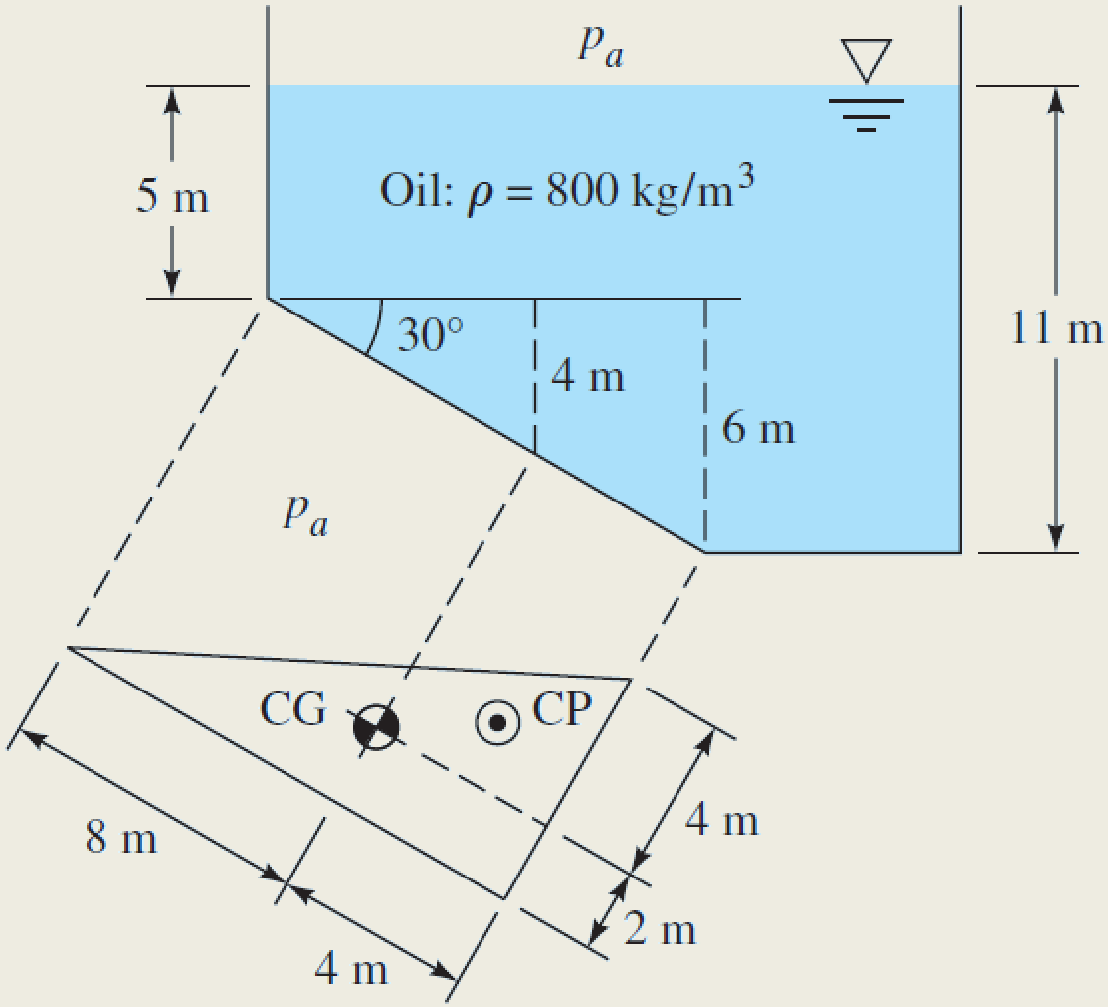
$$B_z = 30{,}700 \text{ lbf}$$

Example 2

A tank of oil has a right-triangular panel near the bottom, omitting p_a find:

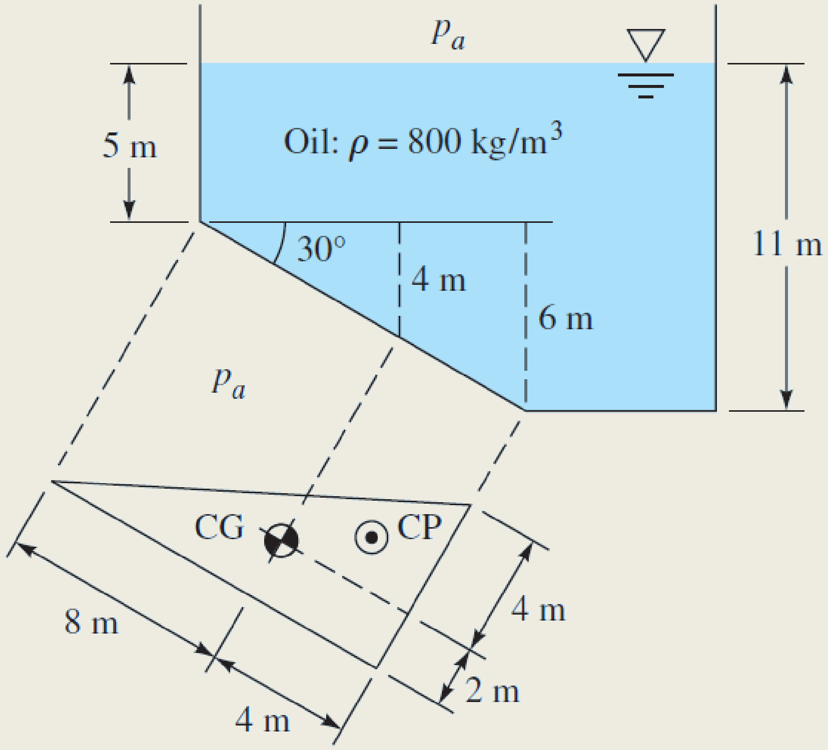
(a) the hydrostatic force

(*b*) the center of pressure on the panel



Example 2

- Part A:



$$A = 0.5 \times 12m \times 6m = 36\, m^2$$

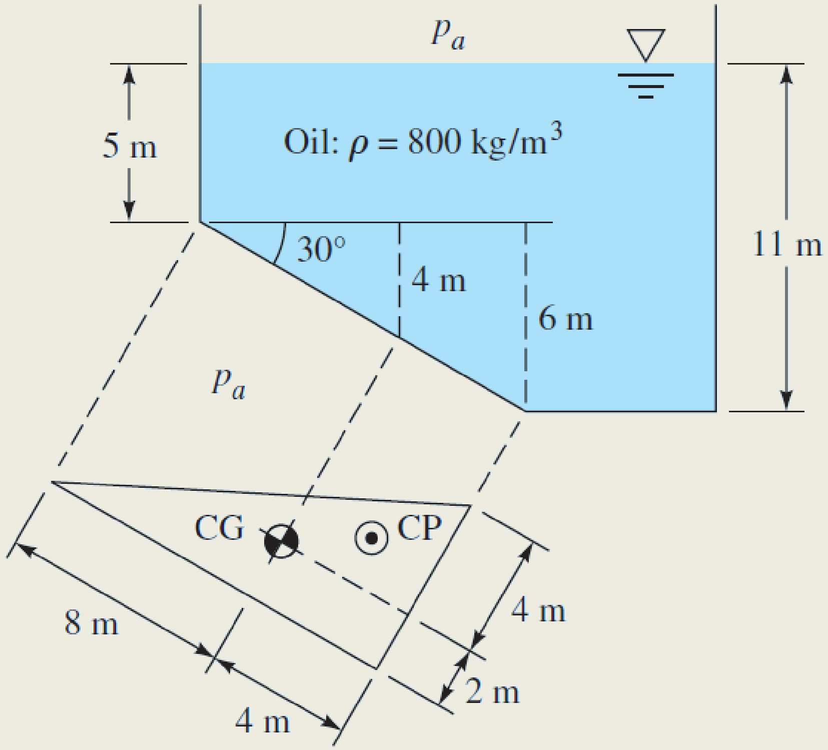
$$h_{CG} = 5 + \frac{2}{3}6 = 9\, m$$

$$F = \rho g h_{CG} A = (800 \text{ kg}/\text{m}^3)(9.807 \text{ m}/\text{s}^2)(9 \text{ m})(36 \text{ m}^2)$$

$$= 2.54 \times 10^6 \,(\text{kg}\cdot\text{m})/\text{s}^2 = 2.54 \times 10^6 \text{ N} = 2.54 \text{ MN}$$

Example 2

- Part B:



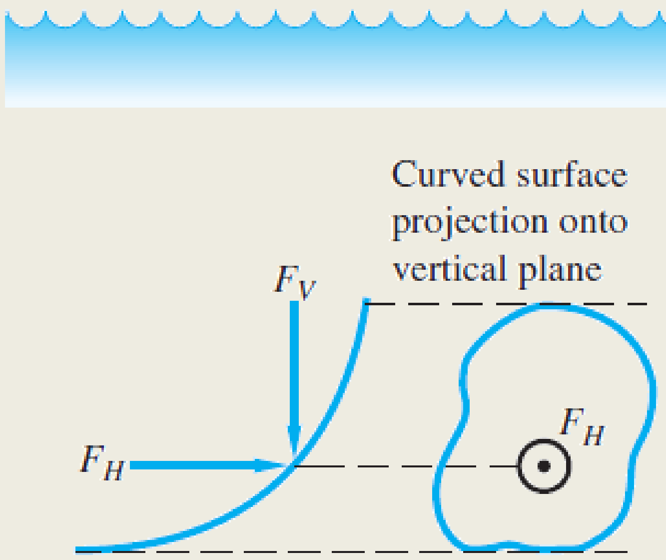
$$I_{xx} = \frac{bL^3}{36} = \frac{(6\text{ m})(12\text{ m})^3}{36} = 288\text{ m}^4$$

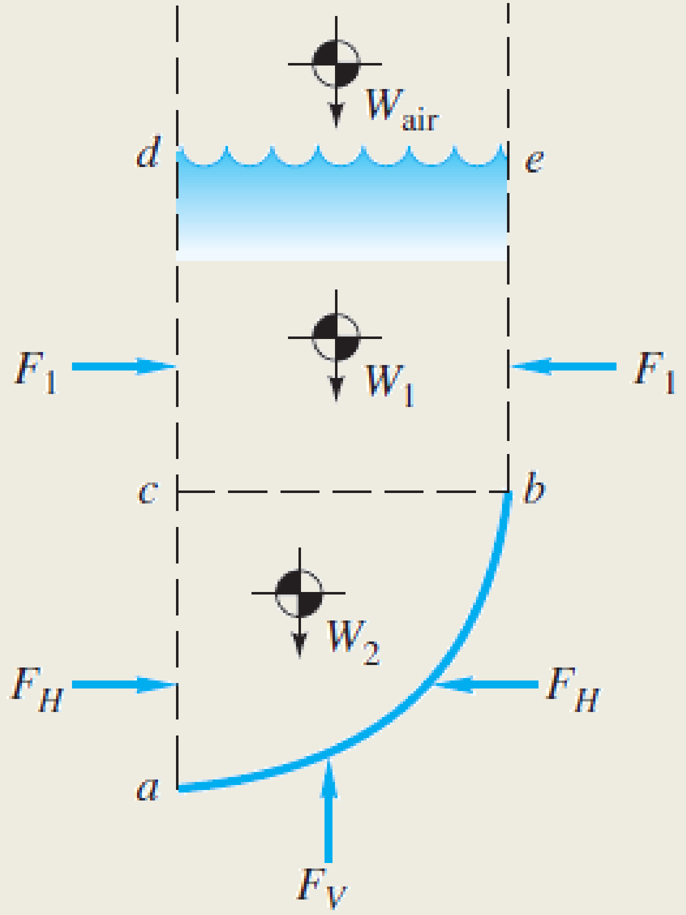
$$I_{xy} = \frac{b(b - 2s)L^2}{72} = \frac{(6\text{ m})[6\text{ m} - 2(6\text{ m})](12\text{ m})^2}{72} = -72\text{ m}^4$$

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(288\text{ m}^4)(\sin 30°)}{(9\text{ m})(36\text{ m}^2)} = -0.444\text{ m}$$

$$x_{CP} = -\frac{I_{xy}\sin\theta}{h_{CG}A} = -\frac{(-72\text{ m}^4)(\sin 30°)}{(9\text{ m})(36\text{ m}^2)} = +0.111\text{ m}$$

Pressure force on curved surfaces





Pressure force on curved surfaces

- **Horizontal component:**

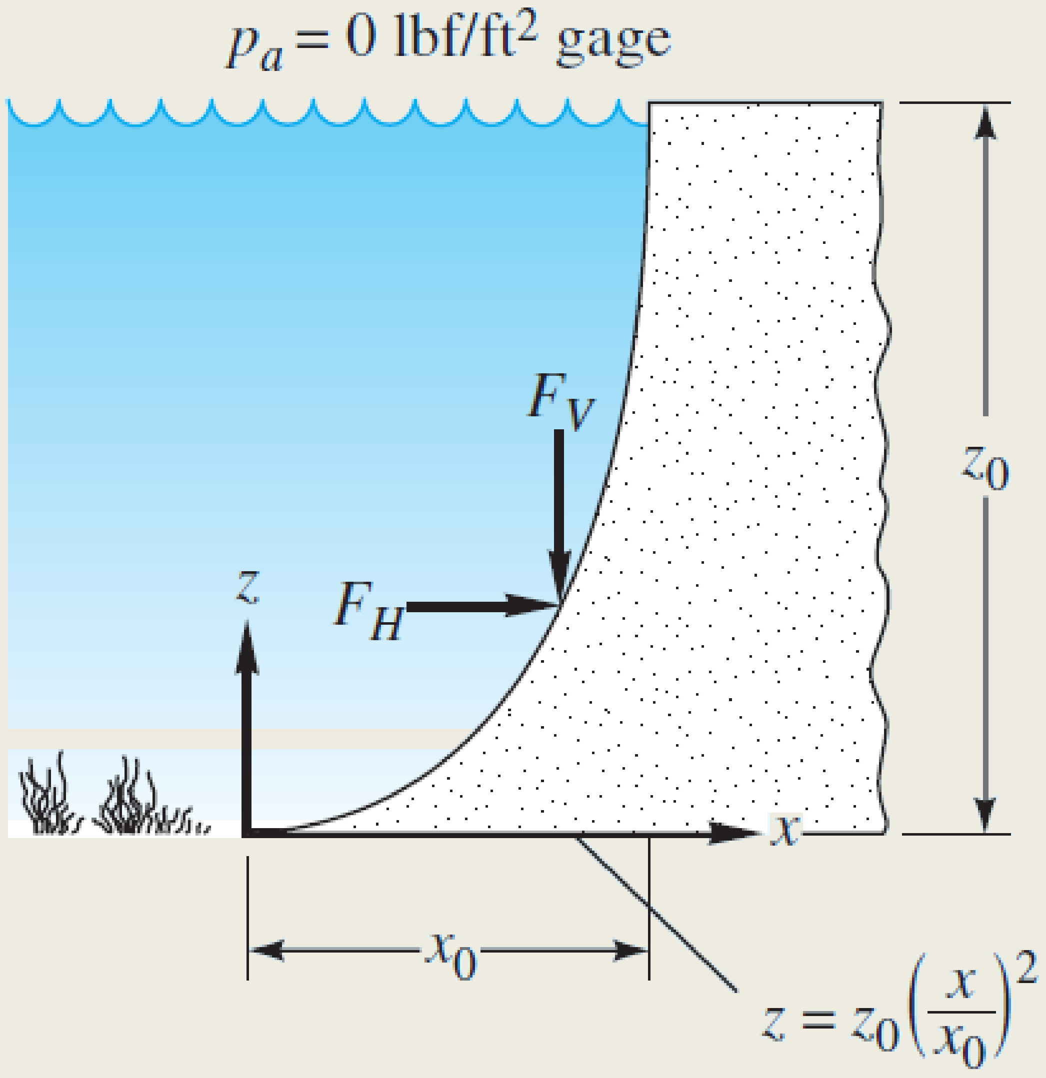
It equals the force on the plane area formed by the projection of the curved surface onto a vertical plane normal to the component.

- **Vertical component:**

It equals (in magnitude and direction) the weight of the entire column of fluid, both liquid and atmosphere, above the curved surface

Example 3

Neglecting p_a, compute the vertical and horizontal forces on the dam with the width of 50 ft (x_0 = 10 ft, z_0 = 24 ft).



Example 3

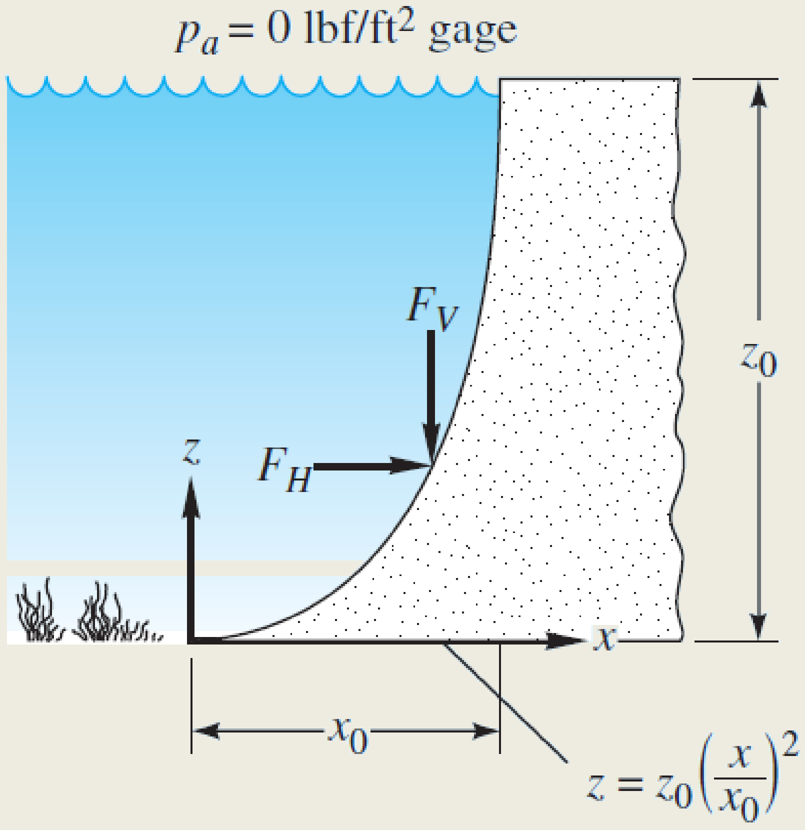
Horizontal force:

h_{CG} = 24/2 = 12 ft

A_{proj} = 50*24 = 1200 ft^2

$$F_H = \gamma h_{CG} A_{proj} = \left(62.4 \frac{\text{lbf}}{\text{ft}^3}\right)(12\text{ ft})(1200\text{ ft}^2) =$$

$$898{,}560\text{ lbf} \approx 899 \times 10^3\text{ lbf}$$



Example 3

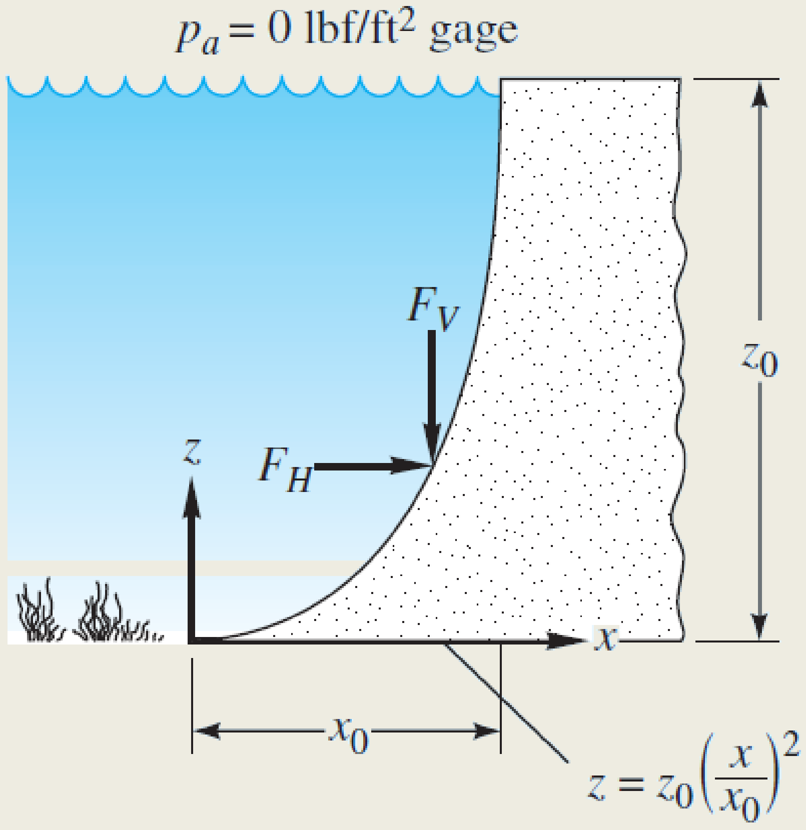
Verctical force

$$Area = z0\,x0 - \left(\int_0^{x0} \frac{z0\,x^2}{x0^2}\,dx\right)$$

$$Area = \frac{2}{3}\,z0\,x0$$

$$F_V = \gamma A_{section} b = \left(62.4\frac{\text{lbf}}{\text{ft}^3}\right)\left[\frac{2}{3}(24\text{ ft})(10\text{ ft})\right](50\text{ ft}) =$$

$$499{,}200\text{ lbf} \approx 499 \times 10^3\text{ lbf}$$

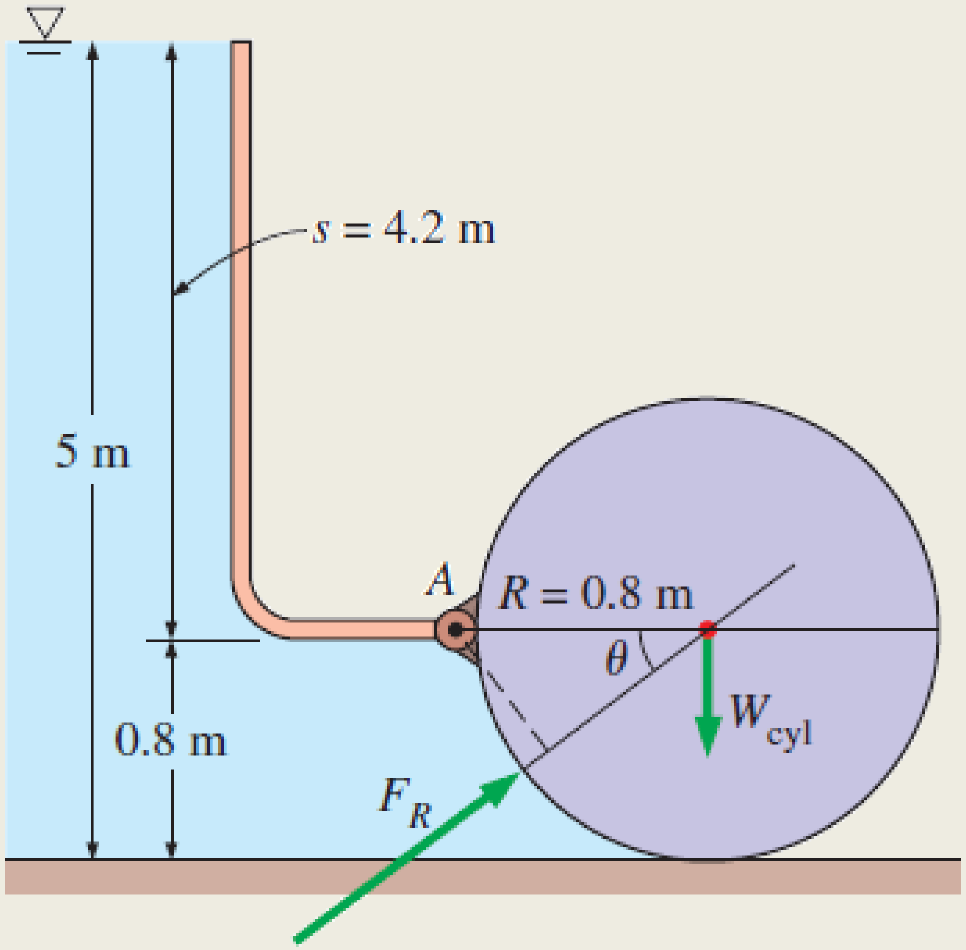


Example 4

A long solid cylinder of radius 0.8 m hinged at point *A* is used as an automatic gate. When the water level reaches 5 m, the gate opens by turning about the hinge at point *A*. Determine:

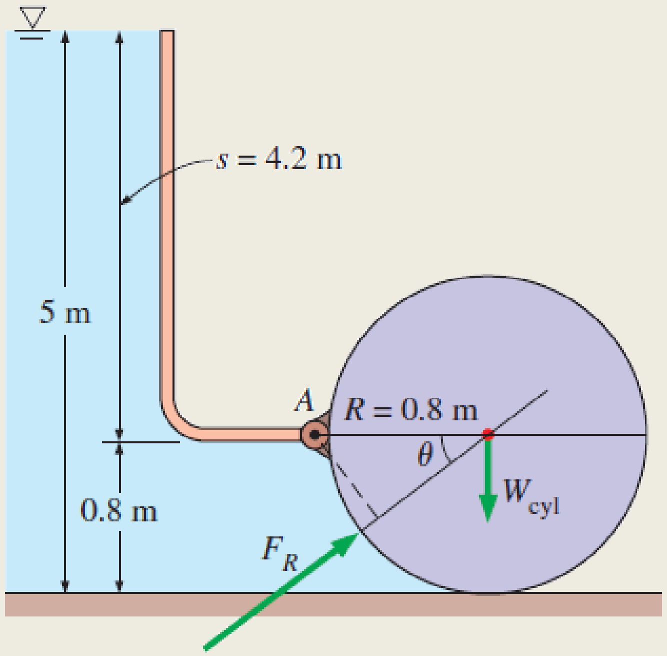
(a) the hydrostatic force acting on the cylinder and its line of action when the gate opens

(b) the weight of the cylinder per m length of the cylinder.



Example 4

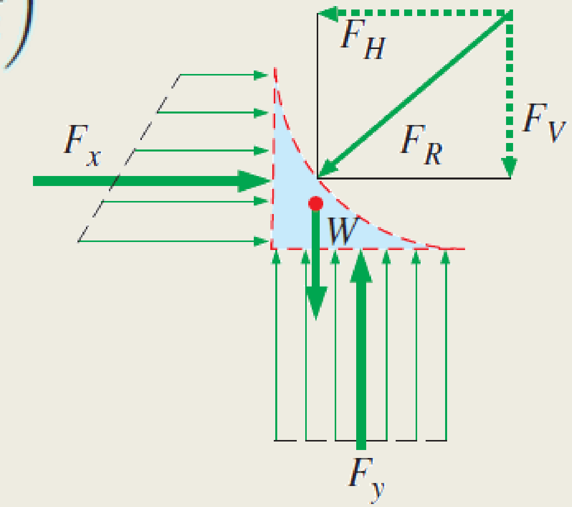
The horizontal force is the pressure force acting on the vertical projection of the cylinder (per 1 m of depth):



$$F_H = F_x = P_{avg} A = \rho g h_C A = \rho g (s + R/2) A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.2 + 0.8/2\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right)$$

$$= 36.1\ \text{kN}$$



Example 4

The vertical force is the weigh of the fluid column above the surface. The volume above the surface is (per 1 m of depth):

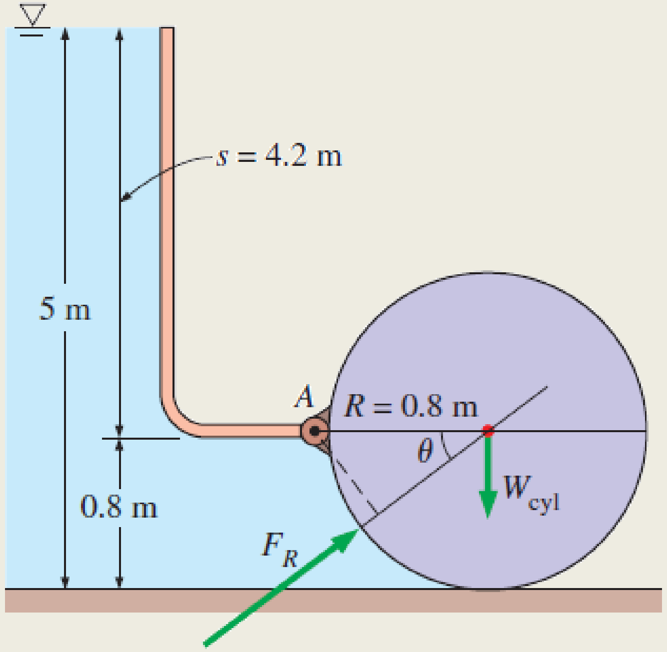
$$\text{Vol} = (4.2 \times 0.8 + \pi/4 \times 0.8^2) \times 1 = 3.8624\ \text{m}^3$$

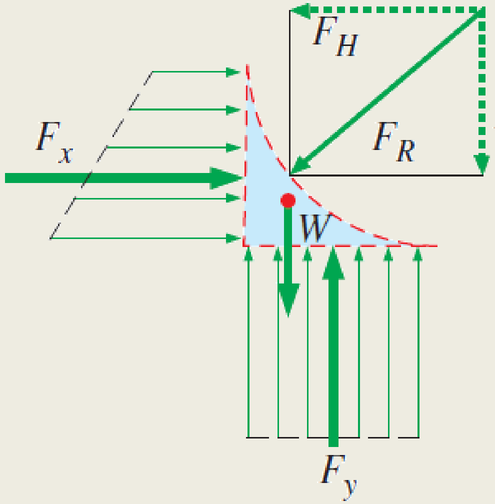
$$F_v = \text{Vol} \times \gamma = 3.8624\ \text{m}^3 \times 1000\ \text{kg/m}^3 \times 9.81\ \text{m/s}^2 =$$

$$37890\ \text{kg.m/s}^2 = 37.89\ \text{kN}$$

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = \mathbf{52.3\ kN}$$

$$\tan\theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4°$$





Example 4

Part (b): taking a moment around point A and equating it to zero (equilibrium):

$$F_R R \sin\theta - W_{cyl} R = 0 \rightarrow$$

$$W_{cyl} = F_R \sin\theta = (52.3 \text{ kN}) \sin 46.4° = \mathbf{37.9 \text{ kN}}$$