MECHANICS OF FLUIDS

Lecture 4 – Buoyancy & rigid body motion

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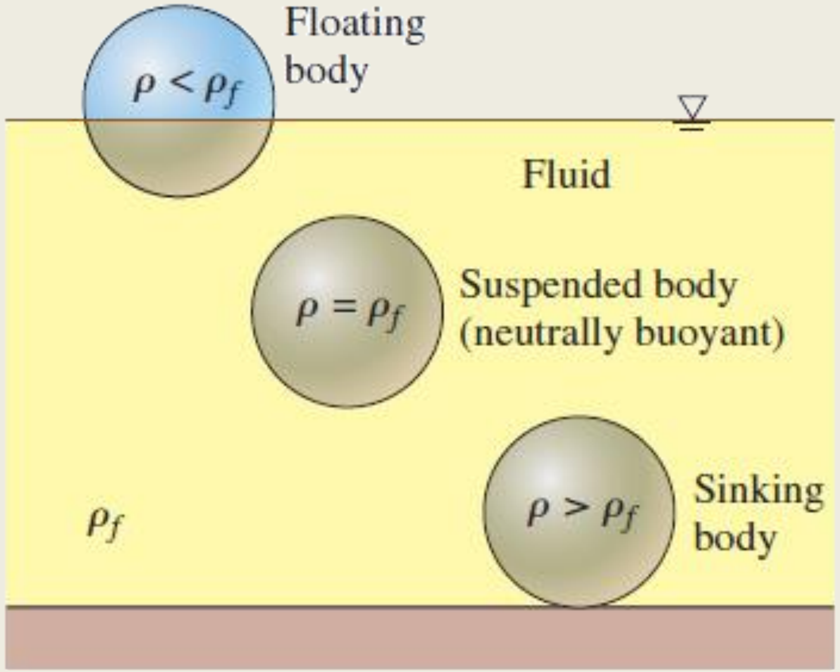
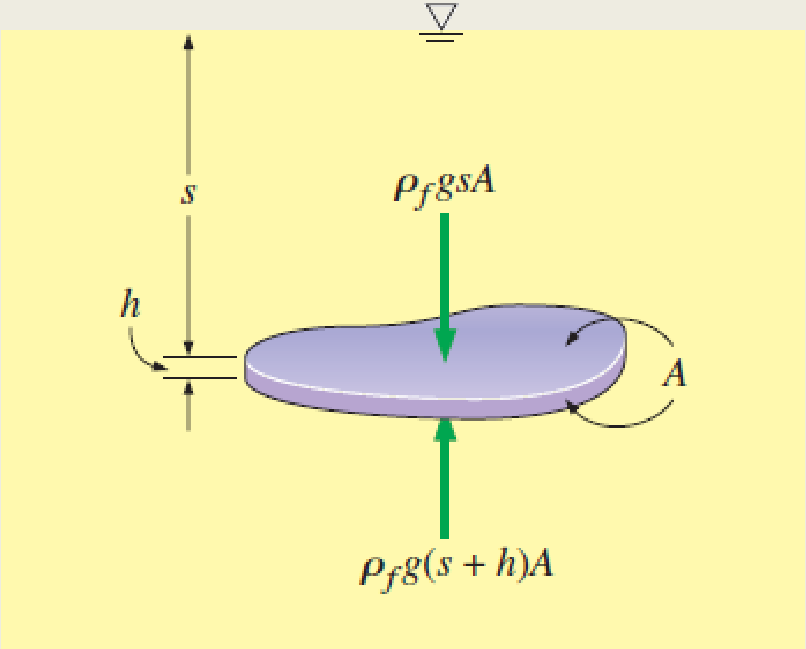
Note

- All the art-work contents of this lecture are obtained from the following sources, unless otherwise stated:
  - *Fluid Mechanics, 8th edition, Frank M. White, McGraw-Hill, 2016.*
  - *Fluid Mechanics: Fundamental and Applications, 3rd edition, Yunus A. Cengel, John M. Cimbala, McGraw-Hill, 2014.*

Buoyancy

- Rules of buoyancy for submerged or floating bodies:
  1. *A body immersed in a fluid experiences a vertical buoyant force equal to* ***the weight of the fluid it displaces****.*
  2. *A floating body* ***displaces its own weight*** *in the fluid in which it floats.*

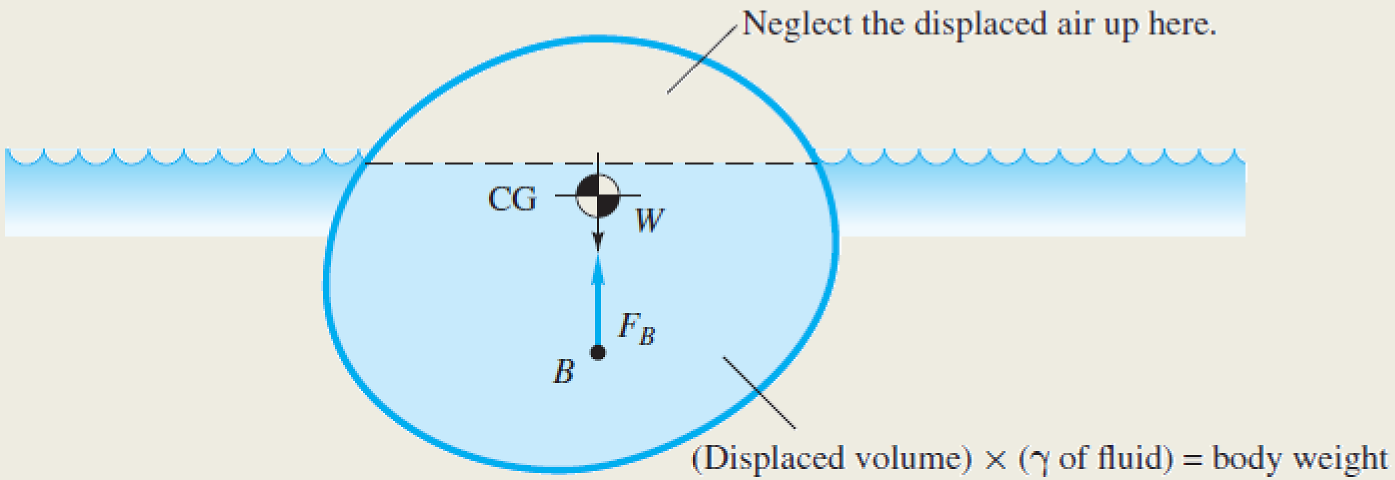




Buoyancy

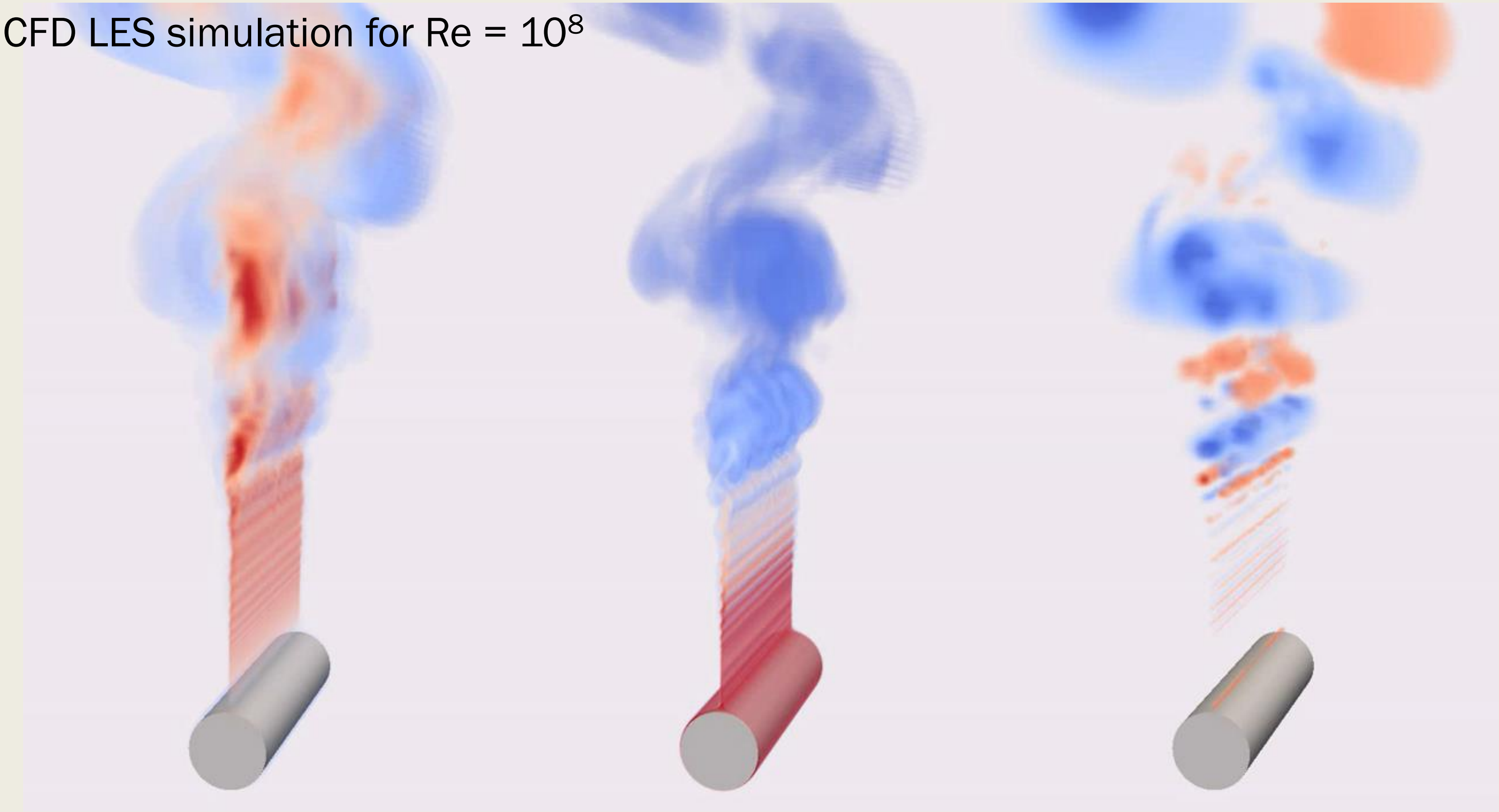
- The **line of action** of buoyant force passes through the **center of volume of the displaced part (center of buoyancy).**
- For multiple layers of fluids:

$$(F_B)_{LF} = \sum \rho_i g(\text{displaced volume})_i$$



Natural heat transfer (heated tube in water)

CFD LES simulation for $Re = 10^8$



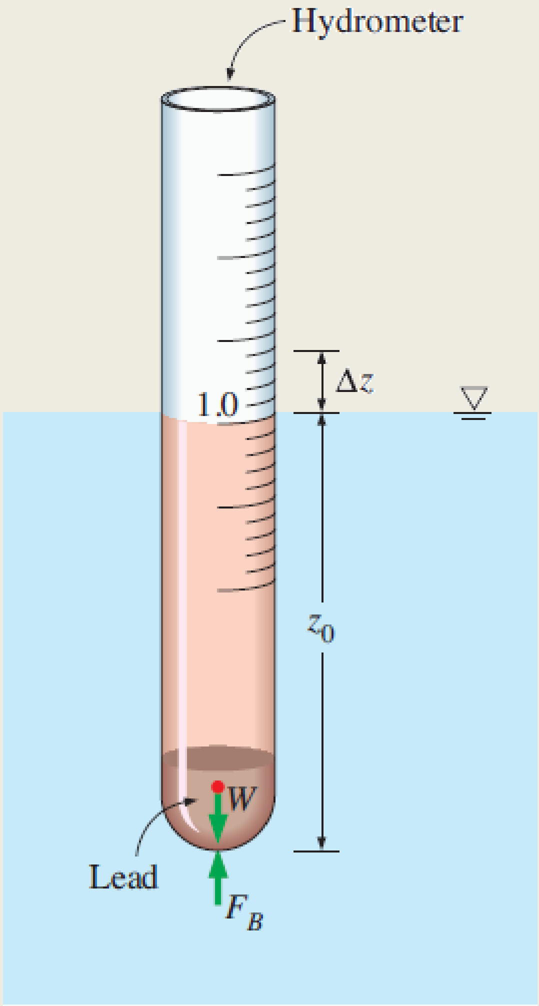
Velocity temperature pressure

source: https://www.youtube.com/watch?v=9thSIO03XRk

Hydrometer

- It is used for measuring the specific gravity of liquids.

- Neglecting the weight of glass, obtain a relation for the specific gravity of a liquid as a function of Δz?



Hydrometer

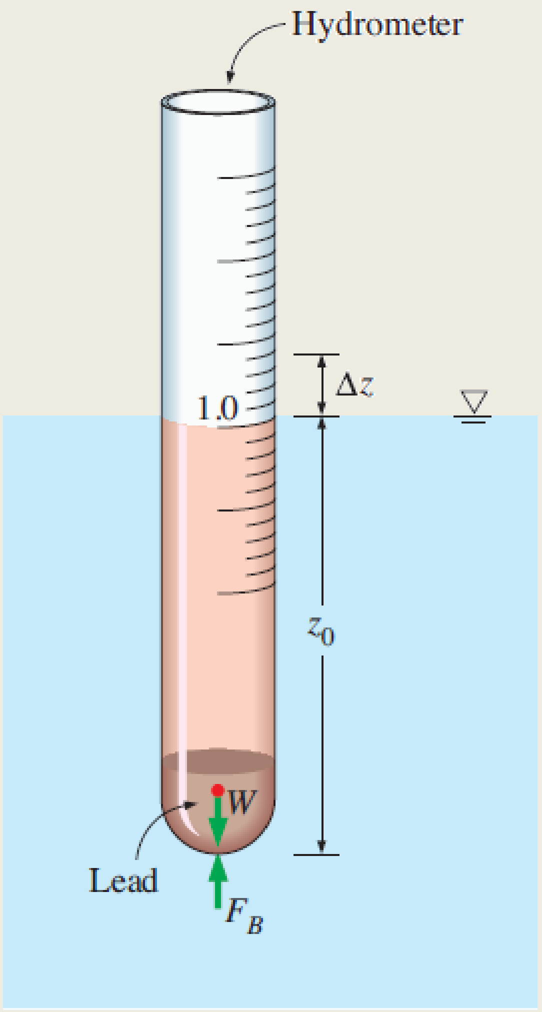
Hydrometer in pure water (at equilibrium):

$$W_{\text{hydro}} = F_{B,w} = \rho_w g V_{\text{sub}} = \rho_w g A z_0$$

- W_{hydro}: Hydrometer weight
- $F_{B,w}$: Buoyant force of water
- A: Cross section of hydrometer
- z_0: Floating distance

Hydrometer in unknown liquid:

$$W_{\text{hydro}} = F_{B,f} = \rho_f g V_{\text{sub}} = \rho_f g A (z_0 + \Delta z)$$



Hydrometer

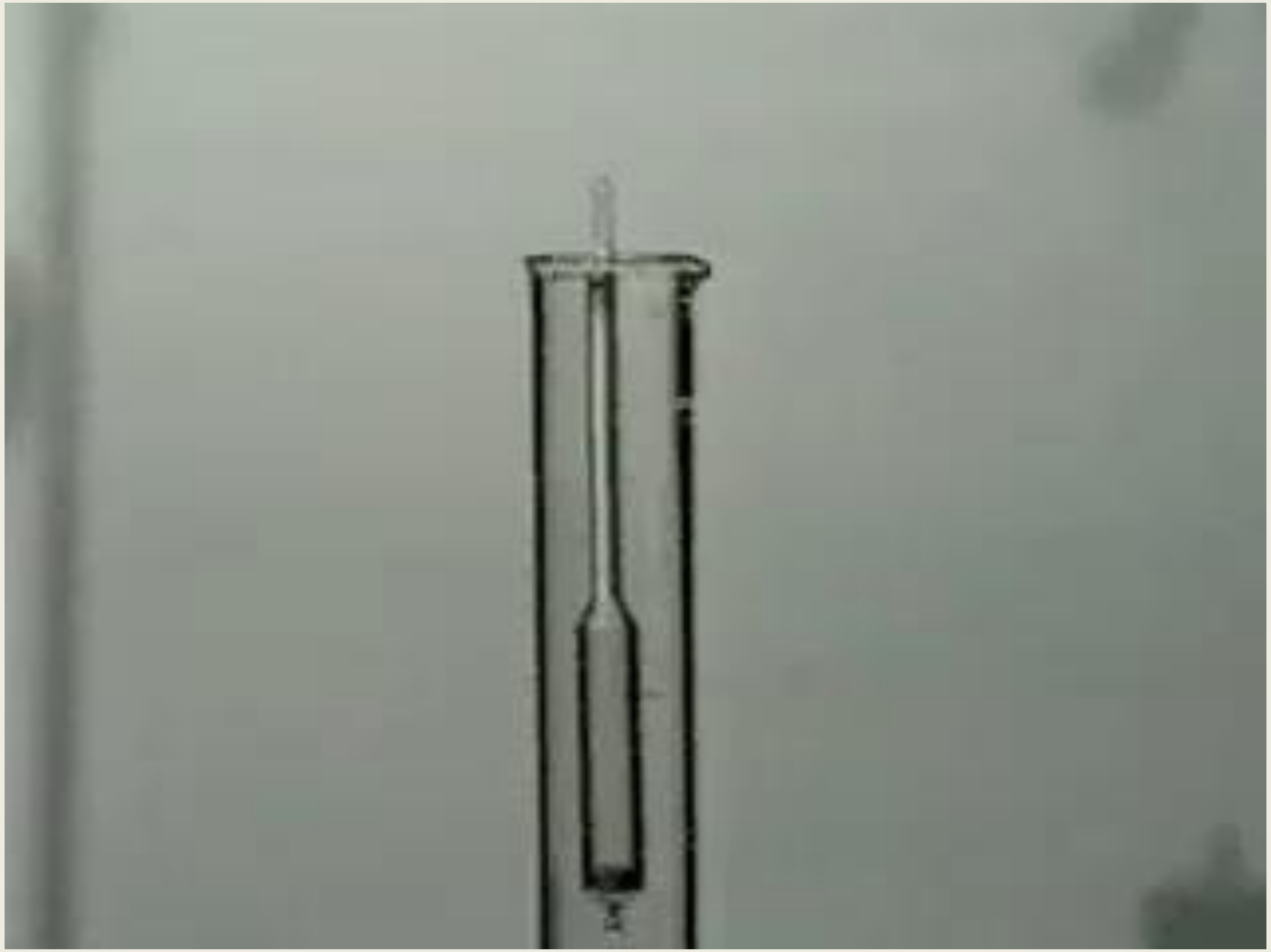
- Equating the two relations:

$$\rho_w g A z_0 = \rho_f g A(z_0 + \Delta z) \quad \rightarrow \quad SG_f = \frac{\rho_f}{\rho_w} = \frac{z_0}{z_0 + \Delta z}$$

- If $\Delta z > 0$, the hydrometer sink into the liquid, the fluid is **lighter** than water.

- If $\Delta z < 0$, the hydrometer stays at a upper level than z_0, the fluid is **heavier** than water.

Hydrometer

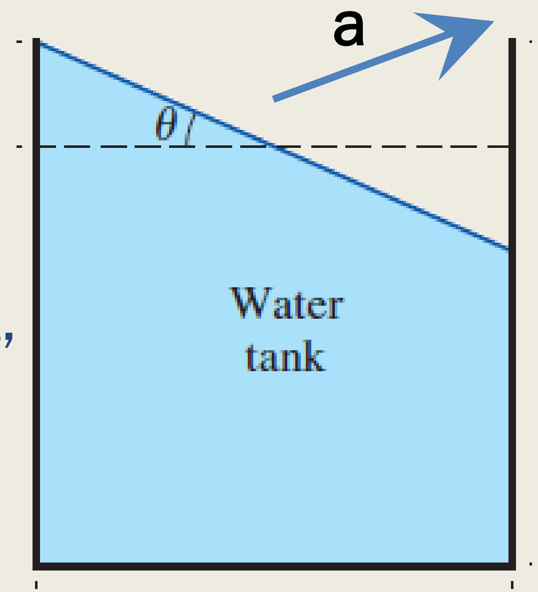


Density column

- Density column is used for precise measurement of solid particles (polymers).
- The density of liquid in the column increases with height from bottom with highest density to the top with lowest.
- The liquid density is adjusted by mixing two miscible liquids with slightly different densities.
- Floaters are used to mark the density along the column height.
- Based on BS2782 Part 6 Method 620D, ASTM D1505-68, ISO 1183.

source: https://www.ametektest.com/products/application-specific-testers/polymer-testers/density-columns

Rigid body motion



- Consider the fluid in a tank and the tank is accelerated in one direction
- After some initial transition, a new free surface, mostly flat, is formed.
- If the entire fluid moves (and accelerated) with the same speed, **no shear stresses exist** in the fluid (because no deformation or strain exists). It is told that the fluid moves like a **rigid body**.
- Recall the basic equation for the fluid element:

$$\sum \mathbf{f} = \mathbf{f}_{\text{press}} + \mathbf{f}_{\text{grav}} + \mathbf{f}_{\text{visc}} = -\nabla p + \rho \mathbf{g} + \mathbf{f}_{\text{visc}} = \rho \mathbf{a}$$

0

Rigid body motion

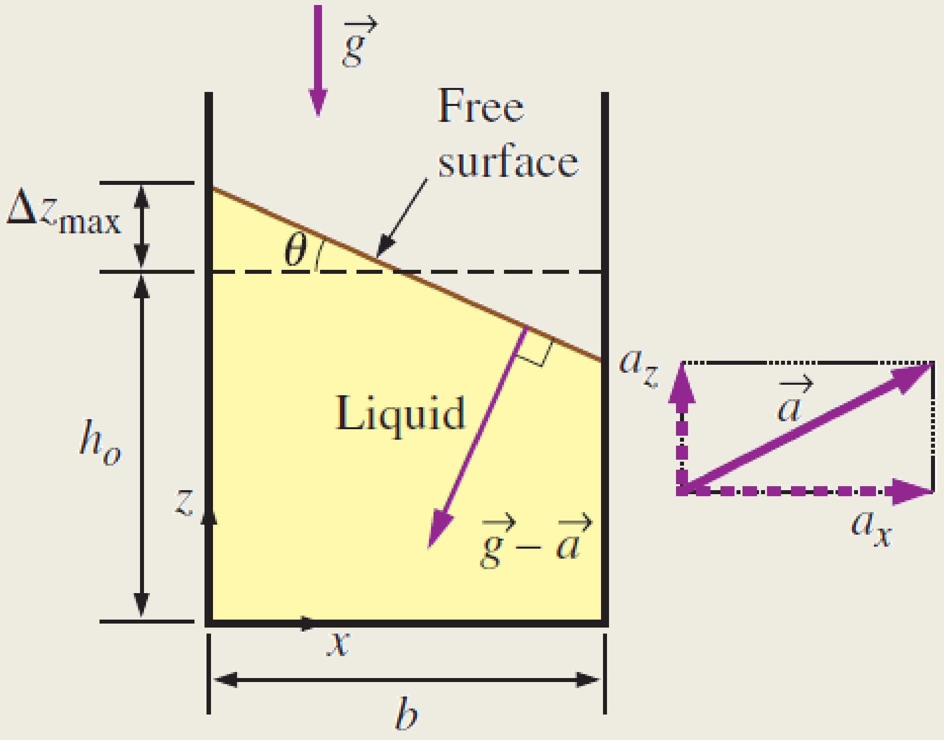
- The equation of motion for the fluid in rigid body motion becomes:

$$\boldsymbol{\nabla} P = \rho(\boldsymbol{g} - \boldsymbol{a})$$

- If The fluid is accelerating on a straight path:

$$\frac{\partial P}{\partial x} = -\rho a_x, \quad \frac{\partial P}{\partial y} = 0$$

$$\frac{\partial P}{\partial z} = -\rho(g + a_z) \qquad (*)$$



- The total differential of P(x,z) is:

$$dP = -\rho a_x\, dx - \rho(g + a_z)\, dz \qquad (**)$$

Rigid body motion

- The pressure distribution is obtained by integrating the differential equations (*):

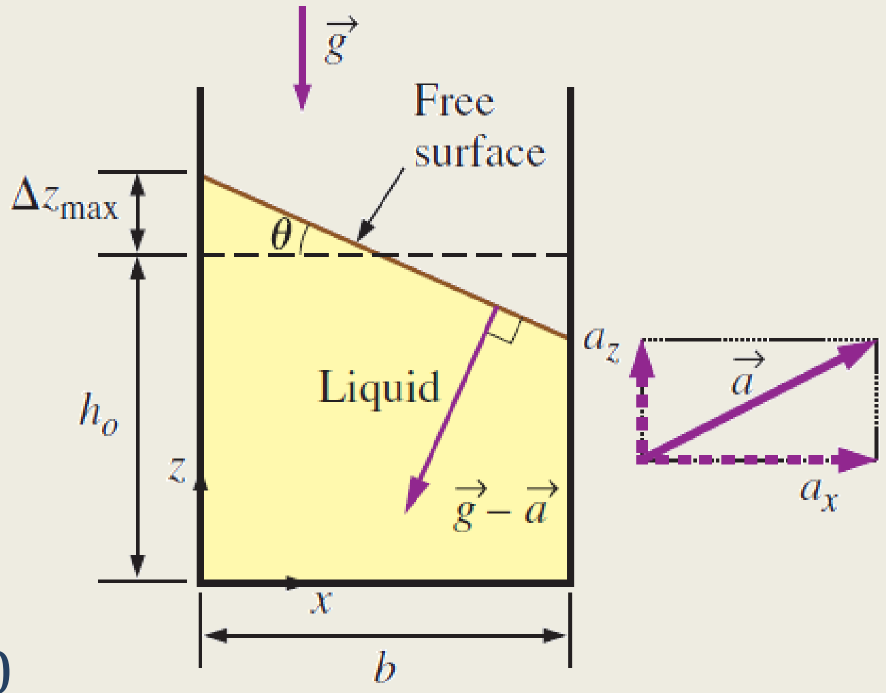
$$P_2 - P_1 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_1)$$

- At Constant pressure surfaces (isobar):

$$dP = 0$$

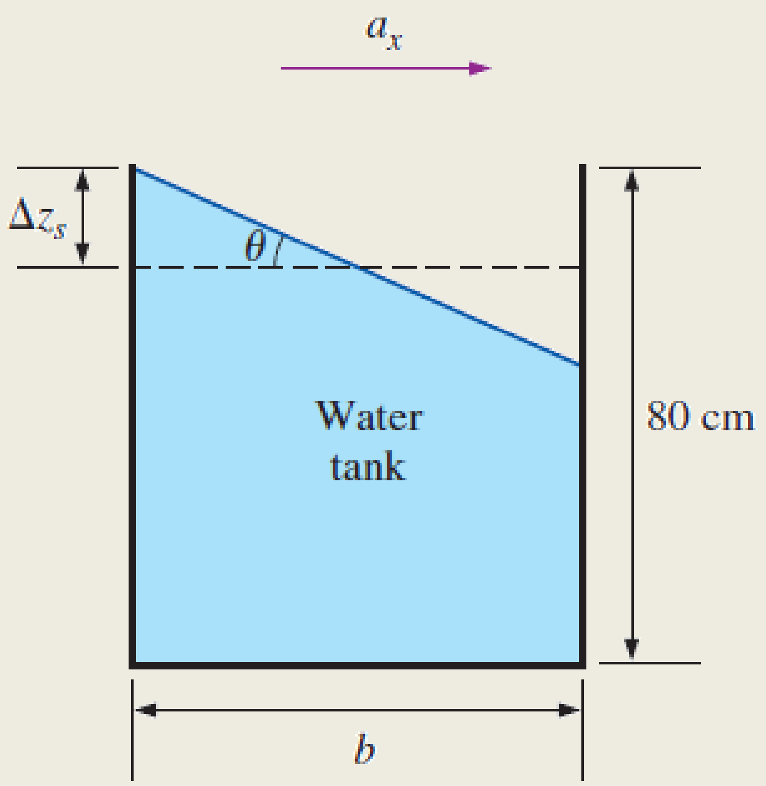
- From (**) we have $-\rho a_x dx - \rho(g + a_z)dz = 0$

$$\text{Slope} = \frac{dz_{\text{isobar}}}{dx} = -\frac{a_x}{g + a_z} = -\tan\theta$$

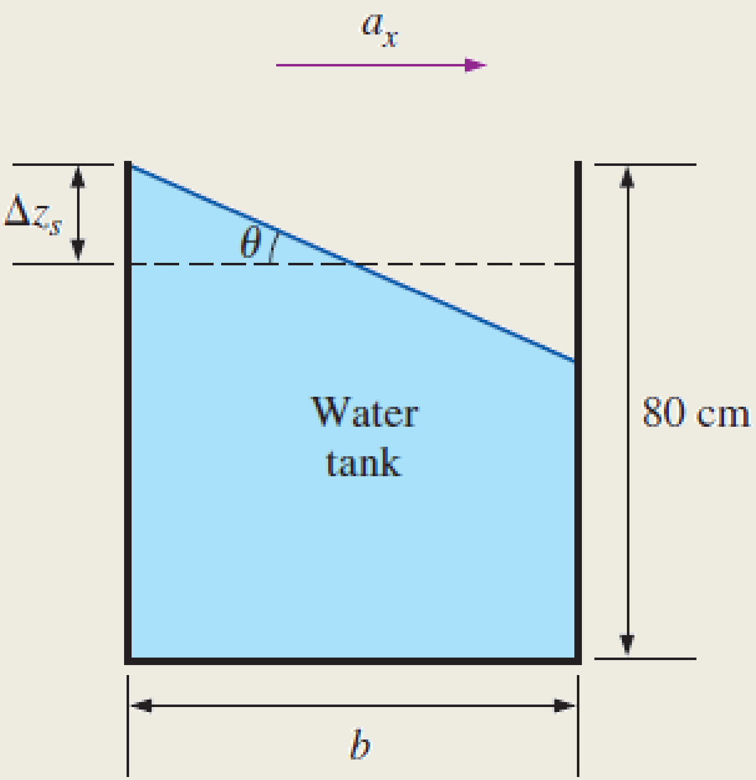


Example

An 80-cm-high tank of cross section 2 m × 0.6 m that is partially filled with water is to be transported. It accelerates from 0 to 90 km/h in 10 s. If it is desired that no water spills during acceleration, determine the allowable initial water height in the tank. Would you recommend the tank to be aligned with the long or short side parallel to the direction of motion?



Example



Assuming the road is horizontal and the acceleration is constant:

$$a_x = \frac{\Delta V}{\Delta t} = \frac{(90 - 0)\text{ km/h}}{10\text{ s}}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 2.5\text{ m/s}^2$$

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{2.5}{9.81 + 0} = 0.255 \qquad (\text{and thus } \theta = 14.3°)$$

The long side is parallel to motion direction

$$\Delta z_{s1} = (b_1/2)\tan\theta = [(2\text{ m})/2] \times 0.255 = 0.255\text{ m} = \mathbf{25.5\text{ cm}}$$

The short side is parallel to motion direction

$$\Delta z_{s2} = (b_2/2)\tan\theta = [(0.6\text{ m})/2] \times 0.255 = 0.076\text{ m} = \mathbf{7.6\text{ cm}}$$